Module 4: Z-scores, Normal Curve, and Simple Probability

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Introduction to Statistics for the Social Sciences

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Outline

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- Knowing the mean of the distribution allows us to tell if a score is above or below the average.
- An individual scores 26 on a leadership test.
  - What does that mean? Are they a good leader or a bad leader?
  - If the mean of the test is 20, then we know that the person is above average in leadership (compared to others who have taken the test).

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Example 1

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- Knowing the standard deviation allows us to tell how much above or below the mean that score is in relations to the spread of the distribution.
- Leadership test: $\overline{X} = 20$, $S = 3$
- The person with the score of 26 is two standard deviations above the mean.

Example 2

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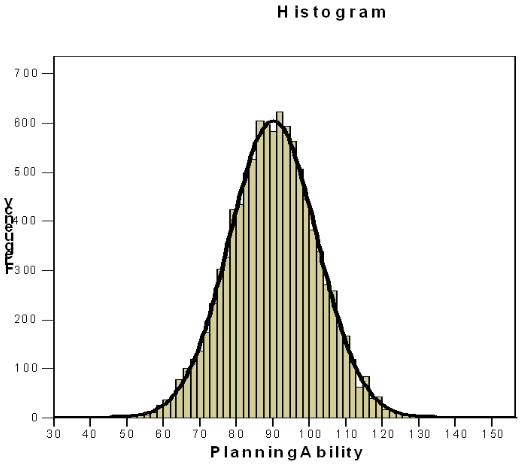
- A person scores 84 on a test of planning ability.
- If $\overline{X} = 90$, $S = 12$, this person's score is 0.5 standard deviations below the mean.
- Thus the person is below average, but not by a lot.

Planning ability: $\overline{X} = 90, S = 12$



We can visualize that the score of 84 would be slightly lower than the mean.

Examples 3 and 4

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- Thus the person is above average by quite a bit.
- What if a person scored 78?
  - Then the person is below the average by about the average amount that scores deviated from the mean.
- Recall, standard deviation *is* the average amount the scores *deviated* from the mean.

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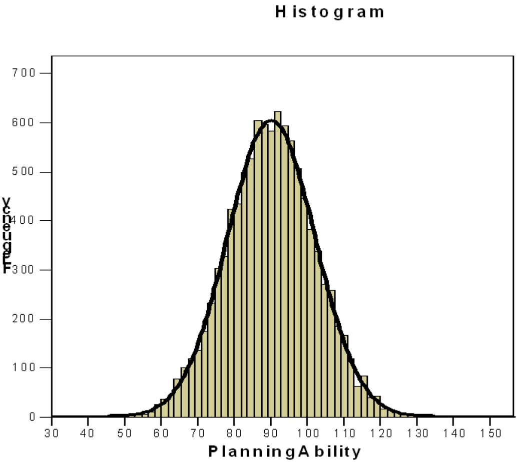
Planning ability: $\overline{X} = 90, S = 12$



Visualize the score of 114 as far above the mean, while the score of 78 would be slightly below the mean.

Z-score

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- The number of standard deviations a particular score is above or below the mean is called its **Z-score**.
  - If a score is below the mean, the Z-score is negative.
  - If a score is above the mean, the Z-score is positive.

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Z-score

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  - So, if $S = 12$, then the average deviation for scores from our Planning Ability test mean is 12.
- The number of standard deviations a particular score is above or below the mean is called its **Z-score**.
  - If a score is below the mean, the Z-score is negative.
  - If a score is above the mean, the Z-score is positive.
- The standard deviation now becomes a kind of yardstick, a unit of measure in its own right.

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Rulers Analogy

The two scales are something like a ruler with inches lined up on the one side and centimeters on the other.

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- A **raw** score of 78 is 1 standard deviation below the mean and so, it has a **Z-score** of -1.

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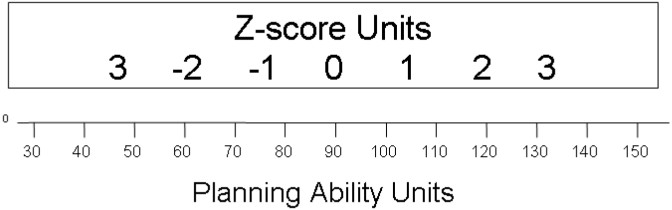
The two scales are something like a ruler with inches lined up on the one side and centimeters on the other.

- A **raw** score of 78 is 1 standard deviation below the mean and so, it has a **Z-score** of -1.
- **Raw scores**: $\overline{X} = 90$, $S = 12$ and **Z-scores**: $\overline{X} = 0$, $S = 1$.



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Z-scores' usefulness

- Z-scores provide a helpful way to compare scores on measures that are on completely different scales.
- If a person scored:
  - 26 on Leadership (example 1: $\overline{X} = 20, S = 3$) and
  - 84 on Planning Ability (example 2: $\overline{X} = 90, S = 12$)
- We can say that person's scores are much higher than average on Leadership and slightly lower than average on Planning Ability.

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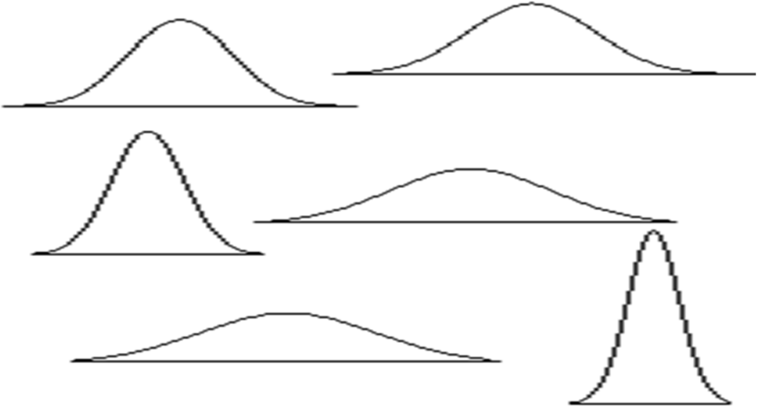
- Example 2: $(-0.5)(12) + 90 = -6 + 90 = 84$
- Example 3: $(2)(12) + 90 = 24 + 90 = 114$
- Example 4: $(-1)(12) + 90 = -12 + 90 = 78$

Normal Curves



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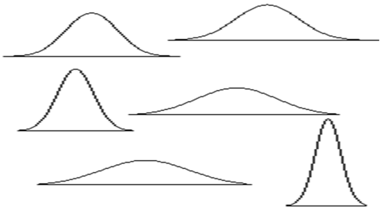
- Symmetrical.
  - Skewness = zero
- Mean, Median, Mode are all the same.
  - At the center.
- Dispersion can change.



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- Precisely defined by:
  - $\overline{X} = 0$
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- So, the shape of the Standard Normal Curve is always the same (i.e., standardized).

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- The scale (x-axis) has no limit above or below the mean (Re: Theoretical Distribution).
- It is always a ‘smooth’ curve because as either a histogram or frequency polygon, the ‘intervals’ of the scale are infinitesimal.

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  - Symmetry; each extreme score in magnitude is as likely as one at the opposite end of the distribution.

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    - Assuming normality allows us to assign probabilities.

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  - If we find the assumption violated, there are often modern (robust) analysis alternatives.

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- The mean of that distribution (of means) will be **the** mean of the population.

http://onlinestatbook.com/simulations/CLT/clt.html

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- Direct random effects: roulette wheel.
  - This has to do with probability, more on this later.

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    - If you know the percentages above and below, you can determine the Z-score.

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- The score itself has no *area*, it is simply a boundary between, a marker if you prefer.
  - Like in geometry, point and line are identifiers.

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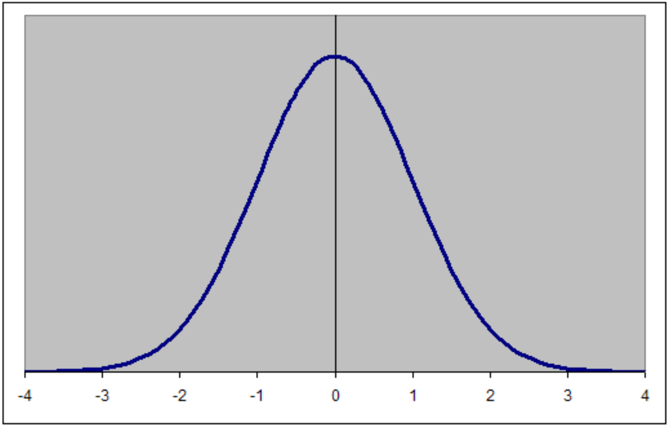

Z-scores along the bottom, the percentage of scores between each can be figured mathematically but, we use tables instead:
http://www.mathsisfun.com/data/standard-normal-distribution-table.html

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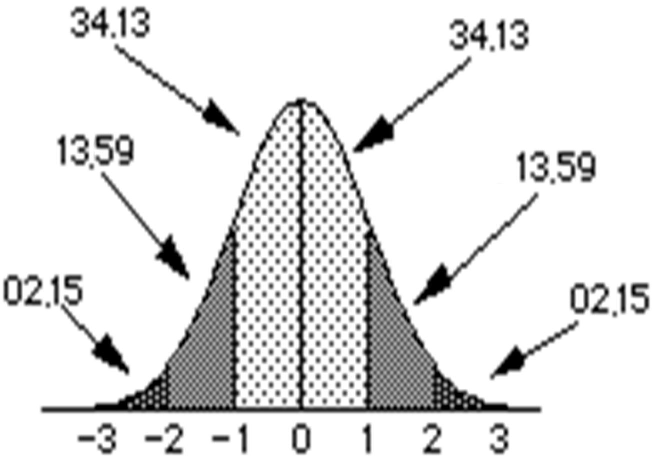

Z-scores along the bottom, percentages around the top

Memorize this image...burn it into your brain!!

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- The proportion of scores between any two Z-scores is the same as the probability of selecting another score between those two scores.
- Keep in mind, the complete table of z-scores and their associated probabilities is available at multiple sites online.

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The *BOTTOM LINE!*

Knowing the formula for converting a raw score to a z-score and knowing the percentages of the standard normal curve (or at least having access to a table):

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Knowing the formula for converting a raw score to a z-score and knowing the percentages of the standard normal curve (or at least having access to a table):

- We can calculate the percentage of scores between any two points of the normal distribution.

The *BOTTOM LINE!*

Knowing the formula for converting a raw score to a z-score and knowing the percentages of the standard normal curve (or at least having access to a table):

- We can calculate the percentage of scores between any two points of the normal distribution.
- For instance, between a raw score and the mean or between any two Z-scores.

More on Percentages.

- When figuring percentage of scores between any two z-scores or between the mean and a Z-score, pay close attention to the column in the table you need.

Z	% Mean to Z	% in Tail
.00	.00	50.00
.01	.40	49.60
.	.	.
.	.	.
.	.	.
.50	19.15	30.85
1.00	34.13	15.87

http://davidmlane.com/hyperstat/z_table.html

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Percentiles = Directional Percentages

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Percentiles = Directional Percentages

- Remember, when finding a percentile rank, the area under the normal curve becomes ranked (from 1 to 100 percentiles).
- For instance, we know that a Z-score of +2 represents 2 standard deviations above the mean. This same location, when converted to a percentile would be the 98th percentile.
  - 50% of the scores are below the mean, (+) 34% of the scores between the mean and a z-score of +1, (+) the other 14% of scores between a Z-score of +1 and a Z-score of +2 = 98% of the scores, or the 98th percentile.

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Percentiles

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Percentiles

- Likewise, a Z-score of -1 which is one standard deviation below the mean would be expressed as the 16th percentile.
  - 2% of the scores are beyond 2 standard deviations below the mean, (+) 14% of the scores between 2 standard deviations below the mean and 1 standard deviation below the mean = 16% of the scores are below our Z-score of -1; a raw score with the Z-score of -1 is the 16th percentile.

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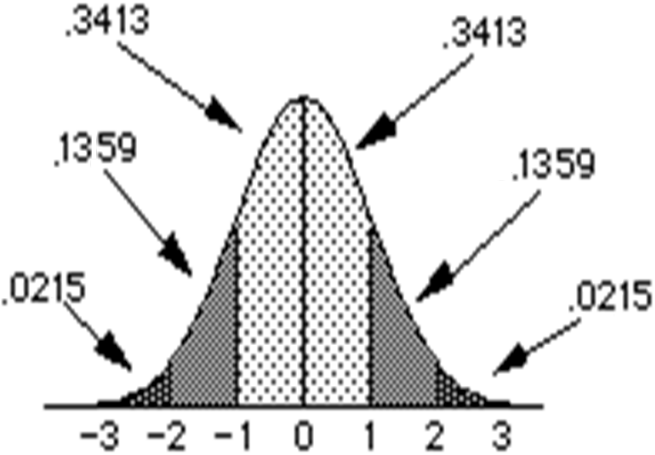

Z-scores along the bottom, probabilities around the top

Memorize this image...burn it into your brain!!

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Percentages & simple Probabilities

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  14% = .1359
  02% = .0215
- The proportion of scores between any two Z-scores is the same as the probability of selecting a case between those two Z-scores.

More on simple Probability

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- Calculating simple probability.
  - The number of possible successful or desirable outcomes divided by the number of **all** possible outcomes.

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    - This is the interpretation we typically use.
- Subjective interpretation of probability.
  - How certain one is that a particular event will happen.
    - There is a 16.66% chance I'll roll a 5 on one independent roll of the die.

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Example simple Probabilities

Remember, simple probability is the number of desired outcomes divided by the number of possible outcomes.

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- Probability of getting a head on one toss of the coin after 2500 tosses of the coin?
    - $p = .50$
    - Remember the old saying: "The coin has no memory."

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Practice Examples

- The ‘Bear Cubs Problem’
- The ‘Balls of Two Colors’ problem

Available toward the bottom of the page at:

http://www.cut-the-knot.org/probability.shtml

Summary of Module 4

Module 4 covered the following topics:

- Describing the position of a score in a distribution.

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All of these topics will be re-visited consistently in future modules.

This concludes Module 4

Next time Module 5.

- Next time we'll begin covering Hypothesis Testing.
- Until next time; have a nice day.

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[1] This document was created in LaTeX using the Beamer package