

Module 6: Examples Handout

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The Research and Statistical Support (RSS) office at the University of North Texas hosts a number of “Short Courses”.

A list of them is available at:

<http://www.unt.edu/rss/Instructional.htm>

Example 1

In Module 5, we had the example research question: Is Scooby's IQ *greater than* that of dogs not in cartoons? We were given the mean and standard deviation of IQ among dogs not in cartoons ($\mu = 100, \sigma = 15$) and we *knew* this population was normally distributed. We used the steps of Null Hypothesis Significance Testing (NHST) to determine if Scooby's Z-score was more extreme than our critical Z-score of 1.64 (a one-tailed test with the significance value of 5% or a p value of .05). We found that Scooby's Z-score was not more extreme than 1.64. Meaning, Scooby's IQ was not significantly greater than non-cartoon dogs.

For this module (6), we have a sample of three cartoon dogs (Scooby, Pluto, & Goofy). We are expanding our research to include a sample of greater than one individual. We are still wondering if cartoon dogs have significantly higher IQ than other dogs (i.e., those not on cartoons). We will work through the steps of NHST to answer our research question.

Scooby scored 123, Pluto scored 145, and Goofy scored 133. Therefore, the mean of our sample mean is:

$$\bar{X} = \frac{\sum X}{n} = \frac{401}{3} = 133.67$$

Step 1: Define the populations and restate the research question as null and alternative hypotheses.

Population 1: Dogs on cartoons.

Population 2: Dogs not on cartoons.

Null hypothesis: $H_0 : \mu_1 = \mu_2$

Alternative hypothesis: $H_1 : \mu_1 > \mu_2$

Step 2: Determine the characteristics of the comparison distribution. This time, the comparison distribution is the **distribution of means** (based on dog IQ scores), because we have a sample of individuals instead of a single individual. We already know the population mean and standard deviation of dogs not in cartoons' IQ ($\mu = 100, \sigma = 15$) and we know that population is normally distributed. So, we then know the mean of the distribution of means ($\mu_M = 100$). However, we must figure the variance and standard deviation using the population of individuals' standard deviation and the number of individuals per sample that made our distribution of means ($n = 3$).

$$\text{Variance of the distribution of means: } \sigma_M^2 = \frac{\sigma^2}{n} = \frac{15^2}{3} = \frac{225}{3} = 75$$

$$\text{Standard deviation of the distribution of means: } \sigma_M = \sqrt{\sigma_M^2} = \sqrt{75} = 8.67$$

Step 3: Determine the cutoff sample score on the comparison distribution at which the null hypothesis should be rejected. This part is exactly like we did previously. Here, we are doing a one-tailed test and using the *usual* 5% cutoff or p value of .05. This results in a Z-score of 1.64 (just like our example in the previous module) for our cutoff or critical sample score.

Step 4: Compute or calculate your sample statistic.

$$Z = \frac{\bar{X} - \mu_M}{\sigma_M} = \frac{133.67 - 100}{8.67} = \frac{33.67}{8.67} = 3.88$$

Step 5: Compare values and make a decision about rejecting the null hypothesis or fail to reject the null hypothesis.

$$Z_{calc} = 3.88 > 1.64 = Z_{crit}$$

So, our Z-calculated is 3.88, which is larger (or more extreme) than our Z-critical of 1.64; meaning that our sample's mean has a Z-score more extreme than our cutoff score. Stated still another way, our sample is more extreme (higher in fact) than 95% of dogs not on cartoons' IQ. We have demonstrated that our sample is somewhere in the top 5%, so we can reject the null hypothesis and conclude that; based on our sample, dogs on cartoons have a higher average IQ than dogs not on cartoons.

Example 2

The second example, we remove Pluto and Goofy; but add UnderDog and Scrappy Doo to our sample. Scooby scored 123, UnderDog scored 109, and Scrappy scored 91. Therefore, the mean of our sample mean is:

$$\bar{X} = \frac{\sum X}{n} = \frac{323}{3} = 107.67$$

Step 1: Define the populations and restate the research question as null and alternative hypotheses.

Population 1: Dogs on cartoons.

Population 2: Dogs not on cartoons.

Null hypothesis: $H_0 : \mu_1 = \mu_2$

Alternative hypothesis: $H_1 : \mu_1 > \mu_2$

Step 2: Determine the characteristics of the comparison distribution.

$$\text{Variance of the distribution of means: } \sigma_M^2 = \frac{\sigma^2}{n} = \frac{15^2}{3} = \frac{225}{3} = 75$$

$$\text{Standard deviation of the distribution of means: } \sigma_M = \sqrt{\sigma_M^2} = \sqrt{75} = 8.67$$

Step 3: Determine the cutoff sample score on the comparison distribution at which the null hypothesis should be rejected. Here, we are doing a one-tailed test and using the *usual* 5% cutoff or p value of .05. This results in the same 1.64 Z-score cutoff.

Step 4: Compute or calculate your sample statistic.

$$Z = \frac{\bar{X} - \mu_M}{\sigma_M} = \frac{107.67 - 100}{8.67} = \frac{7.67}{8.67} = 0.88$$

Step 5: Compare values and make a decision about rejecting the null hypothesis or fail to reject the null hypothesis.

$$Z_{calc} = 0.88 < 1.64 = Z_{crit}$$

So, our Z-calculated is 0.88, which is smaller than our Z-critical of 1.64; meaning that our sample's mean has a Z-score less extreme than our cutoff score. Stated still another way, our sample is not as extreme as the top 5%, so we can not reject the null hypothesis. We conclude that; based on our sample, dogs on cartoons do not have a higher average IQ than dogs not on cartoons. Our sample is not statistically significantly different than dogs not on cartoons.