Module 9: One-Way Analysis of Variance (ANOVA)

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Introduction to Statistics for the Social Sciences

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  - In future modules, we will discuss other analysis of the ANOVA family, such as repeated measures and factorial ANOVA, which features more than one independent variable.

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  - Comparing the stimulant effects (in terms of alertness) of Cocaine, Crystal Meth., Coffee, Red Bull, Mountain Dew, and Placebo.

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- As with the *t* test situation, we evaluate these assumptions (of the populations) using the samples as representatives.

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  - For example, we have a score: $X_{23} = 17$ which indicates this score is located at the second row and the third column; or we could say that 17 is the second score of the third variable because, data are generally arranged in a matrix with the variables as column headings and rows representing cases (i.e. participants).

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  - Stated another way, we seek to discover *mean differences* by analyzing the variances.
- Some authors (textbooks) use 'treatment' instead of 'between' and 'error' instead of 'within'.

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- The ratio of between groups variance to within group variance **is** our F statistic (F_{calc}).

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- Then of course; $MS_w = SOS_w / df_w$

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- The 'numerator degrees of freedom' is df_b while the 'denominator degrees of freedom' is df_w

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  $SOS_t = SOS_b + SOS_w$ and $df_t = df_b + df_w$

ANOVA Summary Table

Table 1: Example ANOVA Summary Table

Source	SOS	df	MS	F_{calc}
Between	64	2	32	11.59
Within	58	21	2.76	
Total	122	23		

Between and Within should add to Total, if not there was an error in computation.

Recall, $SOS/df = MS$ and $MS_b/MS_w = F_{calc}$

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  $F = \frac{Treatment\ differences + Individual\ differences}{Individual\ differences}$

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- If H_0 is not true, then all the groups will be very different.
  - Between groups variance and F will be larger; and likely significant ($p < .05$).
- Conversely; if within groups variance is small, then F will be large and if within groups variance is large, F will be small.
- Remember; $F = MS_b/MS_w$

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- Participants were then instructed to recall as many of the words as possible.

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- Look carefully at the scores.
- What do you notice?

Font Color	Words Recalled
Red	25
Red	22
Red	23
Red	21
Green	10
Green	12
Green	11
Green	13
Blue	18
Blue	17
Blue	19
Blue	16

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Data and Descriptive Statistics

	Red Group	Green Group	Blue Group	Totals
	25	10	18	
	22	12	17	
	23	11	19	
	21	13	16	
$\sum X =$	91	46	70	207
$n =$	4	4	4	$n_t = 12$
$\overline{X} =$	22.75	11.5	17.5	$\overline{X}_{..} = 17.25$
$S =$	1.71	1.29	1.29	
$S^2 =$	2.92	1.66	1.66	6.24
$df =$	3	3	3	$df_t = 11$

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  - Any significant difference among the three means.

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NHST Step 2

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NHST Step 4

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  - Compute your sample statistic(s).
- Calculate *SOS*, *df*, *MS*, and F_{calc} for Between groups, Within groups, and Total (as a check of the other calculations) given the information provided above.

Calculate SOS_b and df_b

Sum of the n of each group times the squared deviations of each group mean from the grand mean.

$$SOS_b = \sum \left[n_j \left(\overline{X}_j - \overline{X}_{..} \right)^2 \right]$$

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$$SOS_b = n_1 \left(\overline{X}_1 - \overline{X}_{..} \right)^2 + n_2 \left(\overline{X}_2 - \overline{X}_{..} \right)^2 + n_3 \left(\overline{X}_3 - \overline{X}_{..} \right)^2$$

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$$SOS_b = 4\left(22.75 - 17.25\right)^2 + 4\left(11.5 - 17.25\right)^2 + 4\left(17.5 - 17.25\right)^2 = 253.50$$

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$$SOS_b = 4\,(22.75 - 17.25)^2 + 4\,(11.5 - 17.25)^2 + 4\,(17.5 - 17.25)^2 = 253.50$$

$$df_b = k - 1 = 3 - 1 = 2$$

Calculate MS_b

Calculation of Mean-Square between, an estimate of the between groups variance:

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Calculation of Mean-Square between, an estimate of the between groups variance:

$$MS_b = \frac{SOS_b}{df_b}$$

$$MS_b = \frac{253.50}{2}$$

$$MS_b = 126.75$$

Calculate SOS_w and df_w

The sum of the squared deviations of each group's score from its group mean.

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$$SOS_w = (25 - 22.75)^2 + (22 - 22.75)^2 + (23 - 22.75)^2 + (21 - 22.75)^2 + (10 - 11.5)^2 + \ldots + (16 - 17.5)^2 = 18.75$$

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$$df_w = n_t - k = 12 - 3 = 9$$

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$$MS_w = \frac{SOS_w}{df_w}$$

$$MS_w = \frac{18.75}{9}$$

$$MS_w = 2.083$$

Calculate SOS_t and df_t

The sum of the squared deviations of each score from the grand mean.

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$$SOS_t = (25 - 17.25)^2 + (22 - 17.25)^2 + (23 - 17.25)^2 + (21 - 17.25)^2 + (10 - 17.25)^2 + \ldots (16 - 17.25)^2 = 272.25$$

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$$df_t = n_t - 1 = 12 - 1 = 11$$

Build the ANOVA Summary Table and Calculate *F*

Table 2: NHST Example ANOVA Summary Table

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Source	*SOS*	*df*	*MS*	F_{calc}
Between	253.50	2	126.75	60.84
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Total	272.25	11		

Build the ANOVA Summary Table and Calculate F

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Source	SOS	df	MS	F_{calc}
Between	253.50	2	126.75	60.84
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Recall, $SOS/df = MS$ and $MS_b/MS_w = F_{calc}$

Build the ANOVA Summary Table and Calculate F

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Recall, $SOS/df = MS$ and $MS_b/MS_w = F_{calc}$

$$F_{calc} = \frac{MS_b}{MS_w} = \frac{126.75}{2.083} = 60.84$$

Notice that $SOS_b + SOS_w = SOS_t$ and $df_b + df_w = df_t$

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Compare and make a decision.

- Decide whether to reject H_0
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  We would reject the null hypothesis and conclude that there was a significant difference among the sample means. This study suggests that there is a difference among the three theoretical populations (each represented by a sample here).
- However, we do not yet know where the significant difference is located, and there may be more than one significant difference among the three groups.

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  - Traditionally, only done when the 'omnibus' F is significant.
  - Reality; almost always done to explore pair-wise differences.

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    - e.g., Orthogonal contrasts.

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- But, we use the same SOS_w and df_w which leads to the same $MS_w = 2.083$
- Then, for each comparison the usual steps are involved: $F = MS_b/MS_w$
- You may wish to control for family-wise error rate inflation by dividing the significance level by the number of comparisons.

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  - One to evaluate $H_{02} : \mu_{green} = \mu_{blue}$ and $H_{12} : \mu_{green} < \mu_{blue}$

Simple Comparisons Example

- Recall, we hypothesized (prior to data collection) the Red group would recall more words than the Blue group and the Green group would recall less words than the Blue group.
- This necessitates two planned comparisons.
  - One to evaluate $H_{01} : \mu_{red} = \mu_{blue}$ and $H_{11} : \mu_{red} > \mu_{blue}$
  - One to evaluate $H_{02} : \mu_{green} = \mu_{blue}$ and $H_{12} : \mu_{green} < \mu_{blue}$
- Using $df_b = 1$ and $df_w = 9$ and with a significance level of 0.05, we get $F_{crit} = 5.12$ for each comparison.

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Planned Comparison 1: Red ($\overline{X} = 22.75$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 20.125$ $\quad df_b = k - 1 = 2 - 1 = 1$

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$$SOS_b = \sum n_j \left(\overline{X}_j - \overline{X}_{..}\right)^2$$

$$SOS_b = 4\,(22.75 - 20.125)^2 + 4\,(17.50 - 20.125)^2 = 55.12$$

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$$\overline{X}_{..} = 20.125 \qquad df_b = k - 1 = 2 - 1 = 1$$

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$$SOS_b = 4\,(22.75 - 20.125)^2 + 4\,(17.50 - 20.125)^2 = 55.12$$

$$MS_b = SOS_b / df_b = 55.12/1 = 55.12$$

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SOS_w and df_w are the same as was used for the omnibus F, so MS_w is the same as well: 2.083

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$MS_b = SOS_b / df_b = 55.12/1 = 55.12$

SOS_w and df_w are the same as was used for the omnibus F, so MS_w is the same as well: 2.083

$F_{calc} = MS_b / MS_w = 55.12/2.083 = 26.46$

Planned Comparison 1: Red ($\overline{X} = 22.75$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 20.125 \qquad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left(\overline{X}_j - \overline{X}_{..}\right)^2$

$SOS_b = 4\,(22.75 - 20.125)^2 + 4\,(17.50 - 20.125)^2 = 55.12$

$MS_b = SOS_b / df_b = 55.12/1 = 55.12$

SOS_w and df_w are the same as was used for the omnibus F, so MS_w is the same as well: 2.083

$F_{calc} = MS_b / MS_w = 55.12/2.083 = 26.46$

So; $F_{calc} = 26.46 > 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Red group recalled significantly more words than the Blue group.

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Planned Comparison 2: Green ($\overline{X} = 11.50$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 14.50$ $\quad df_b = k - 1 = 2 - 1 = 1$

Planned Comparison 2: Green ($\overline{X} = 11.50$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 14.50 \qquad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left(\overline{X}_j - \overline{X}_{..}\right)^2$

Planned Comparison 2: Green ($\overline{X} = 11.50$) versus Blue ($\overline{X} = 17.50$)

$$\overline{X}_{..} = 14.50 \qquad df_b = k - 1 = 2 - 1 = 1$$

$$SOS_b = \sum n_j \left(\overline{X}_j - \overline{X}_{..}\right)^2$$

$$SOS_b = 4\,(11.50 - 14.50)^2 + 4\,(11.50 - 14.50)^2 = 72.0$$

Planned Comparison 2: Green ($\overline{X} = 11.50$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 14.50 \qquad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left(\overline{X}_j - \overline{X}_{..}\right)^2$

$SOS_b = 4\,(11.50 - 14.50)^2 + 4\,(11.50 - 14.50)^2 = 72.0$

$MS_b = SOS_b / df_b = 72.00/1 = 72.00$

Planned Comparison 2: Green ($\overline{X} = 11.50$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 14.50 \qquad df_b = k - 1 = 2 - 1 = 1$

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$MS_b = SOS_b / df_b = 72.00/1 = 72.00$

SOS_w and df_w are the same as was used for the omnibus F, so MS_w is the same as well: 2.083

Planned Comparison 2: Green ($\overline{X} = 11.50$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 14.50 \qquad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left(\overline{X}_j - \overline{X}_{..}\right)^2$

$SOS_b = 4\,(11.50 - 14.50)^2 + 4\,(11.50 - 14.50)^2 = 72.0$

$MS_b = SOS_b / df_b = 72.00/1 = 72.00$

SOS_w and df_w are the same as was used for the omnibus F, so MS_w is the same as well: 2.083

$F_{calc} = MS_b / MS_w = 72.00/2.083 = 34.57$

Planned Comparison 2: Green ($\overline{X} = 11.50$) versus Blue ($\overline{X} = 17.50$)

$\overline{X}_{..} = 14.50 \qquad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left(\overline{X}_j - \overline{X}_{..}\right)^2$

$SOS_b = 4\,(11.50 - 14.50)^2 + 4\,(11.50 - 14.50)^2 = 72.0$

$MS_b = SOS_b / df_b = 72.00/1 = 72.00$

SOS_w and df_w are the same as was used for the omnibus F, so MS_w is the same as well: 2.083

$F_{calc} = MS_b / MS_w = 72.00/2.083 = 34.57$

So; $F_{calc} = 34.57 > 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Green group recalled significantly fewer words than the Blue group.

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- Some Post-hoc tests have embedded controls for *FW* error rates.

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  - where $\overline{X}_l$ refers to the largest mean among the groups, $\overline{X}_s$ the smallest, and n_g is the number of individuals per group.

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    - Which can replace the omnibus F, but with less statistical power.

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- So, if a mean difference ($\overline{X}_i - \overline{X}_j$) is larger than 2.85 we would conclude there is a significant difference between those two means.

Tukey's HSD continued

- With our minimum significant difference calculated at 2.85; we can compare the differences among our three means to it, to determine which means differ *significantly* from the others.

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$q_{r,.05} = 2.85$	11.50	17.50	22.75
Green = 11.50	0	6.00	11.25
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- So, we find a significant difference between each pair of means because, each difference is greater than 2.85.

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- The *df* and the critical difference between means $(\overline{X}_i - \overline{X}_j)$ are modified for inclusion of samples sizes and variances.

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- where subscript *i* and subscript *j* identify descriptive statistics from each group being compared.

Example df'

As an example let's consider the Red and Green groups.

- Red (i): $\overline{X} = 22.75, S^2 = 2.92, n = 4$
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  - See the supplemental handout for an example of the complete, step-by-step calculation.

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- So, given $r = 3$, significance level of 0.05, and now $df' = 5.787$ we look to the q table and find: $q_{crit} \approx 4.60$

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  - Therefore, the Games-Howell test is highly recommended.

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Effect Size Measures for ANOVA

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- Well, it turns out that ANOVA is a member of the **General Linear Model** (GLM) and therefore, both types of effect size can be utilized in the ANOVA situation.
  - GLM will be discussed in more detail in future modules (e.g., regression) but, suffice to say; we have specified a *linear model* here, which generally refers to a linear relationship between the variables we are analyzing.

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- Partial eta-squared is a popular effect size for ANOVA; although it is generally reserved for more complex designs than the one-way situation.
  - Same as η^2 in the one-way situation.
- The **recommended** effect size, for the one-way situation and more complex situations, is Omega squared (ω^2)
  - Omega squared is virtually unbiased.

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Within	18.750	9	2.083	
Total	272.25	11		

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- Which suggests 93.11% of the variance in words recalled (Dependent variable [DV])is attributable to the differences in font color (Indpendent variable [IV]).

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  - This is the same effect size we got for η^2

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$$\omega^2 = \frac{SOS_b - (k-1)MS_w}{SOS_t + MS_w} = \frac{253.5 - (3-1)2.083}{272.25 + 2.083} = \frac{249.332}{274.33} = .9089$$

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- Here we see a less biased, more accurate representation of the relationship between the IV and the DV.
- 90.89% of the variance in words recalled (DV)is attributable to the differences in font color (IV).
- Please note; Omega squared should also be calculated on the results of Planned Comparisons if they were performed (examples on next slide).

Omega squared for planned comparisons

- For Planned comparison 1 from earlier:

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$$\omega^2 = \frac{SOS_b - (k-1)MS_w}{SOS_t + MS_w} = \frac{55.12 - (2-1)2.083}{272.25 + 2.083} = \frac{53.037}{274.33} = .1933$$

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- So, 25.49% of the variance in words recalled is attributable to the difference between Green and Blue font color.

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  - Recall the general equations for CI limits:

$$UL = (+crit) * (SE) + mean$$
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CI for Planned Comparisons

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Adjustments for square root of a negative

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CI for Example Planned Comparisons (PC)

- CI for PC 1: Red ($\overline{X} = 22.75$) vs. Blue ($\overline{X} = 17.50$)

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If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.52 and 1.98.

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$$UL = (+q_{r=3,.05}) + \overline{X}_i - \overline{X}_j = +2.85 + 5.25 = 8.10$$
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- Continued on next slide.

CI for Tukey's HSD PH continued

- CI for PH 2: Red ($\overline{X} = 22.75$) vs. Green ($\overline{X} = 11.50$)

CI for Tukey's HSD PH continued

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- CI for PH 3: Blue ($\overline{X} = 17.50$) vs. Green ($\overline{X} = 11.50$)

$$UL = (+q_{r=3,.05}) + \overline{X}_i - \overline{X}_j = +2.85 + 6.00 = 8.85$$
$$LL = (-q_{r=3,.05}) + \overline{X}_i - \overline{X}_j = -2.85 + 6.00 = 3.15$$

CI for Tukey's HSD PH continued

- CI for PH 2: Red ($\overline{X} = 22.75$) vs. Green ($\overline{X} = 11.50$)

$$UL = (+q_{r=3,.05}) + \overline{X}_i - \overline{X}_j = +2.85 + 11.25 = 14.10$$
$$LL = (-q_{r=3,.05}) + \overline{X}_i - \overline{X}_j = -2.85 + 11.25 = 8.40$$

- If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Green groups to be between 14.10 and 8.40.
- CI for PH 3: Blue ($\overline{X} = 17.50$) vs. Green ($\overline{X} = 11.50$)

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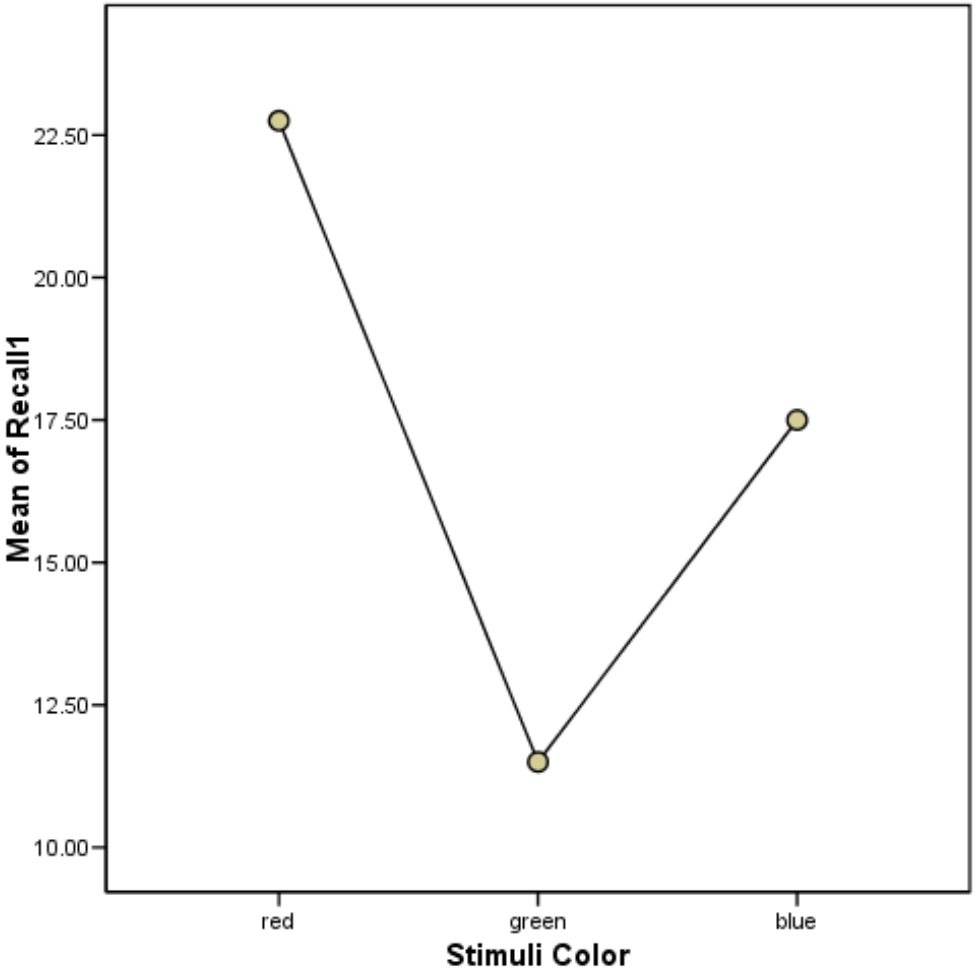

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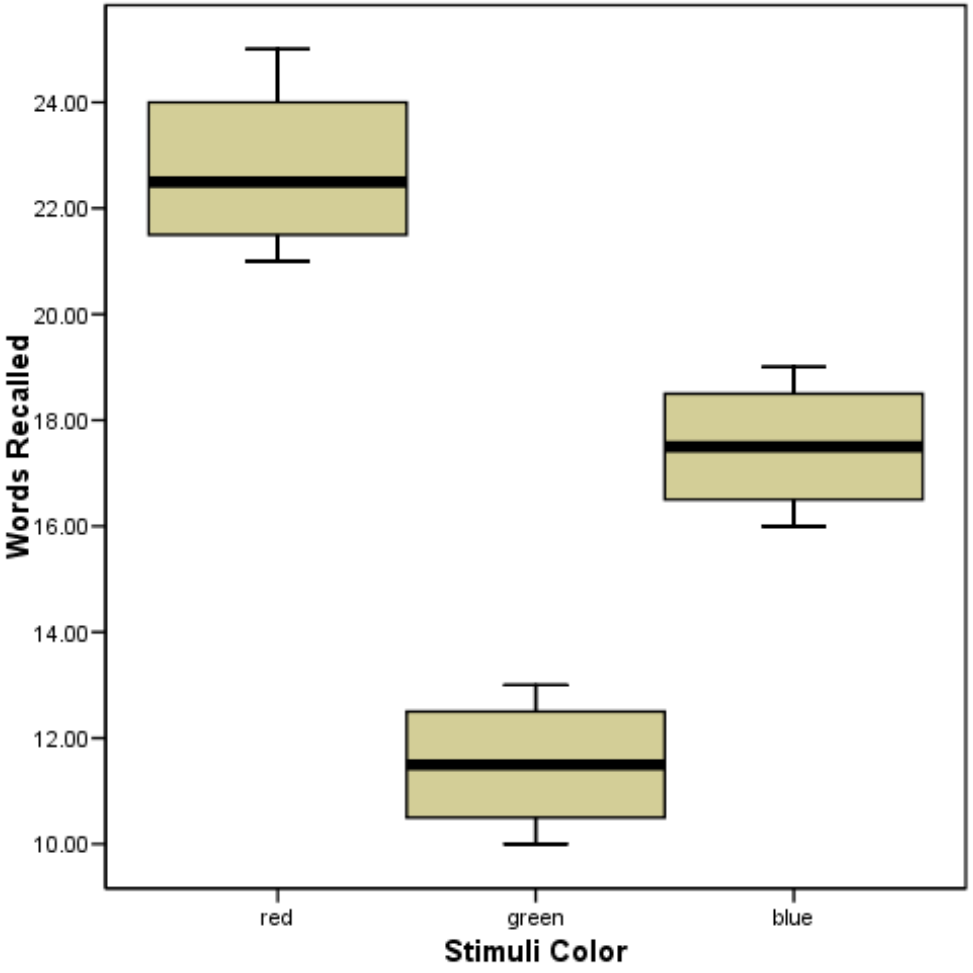

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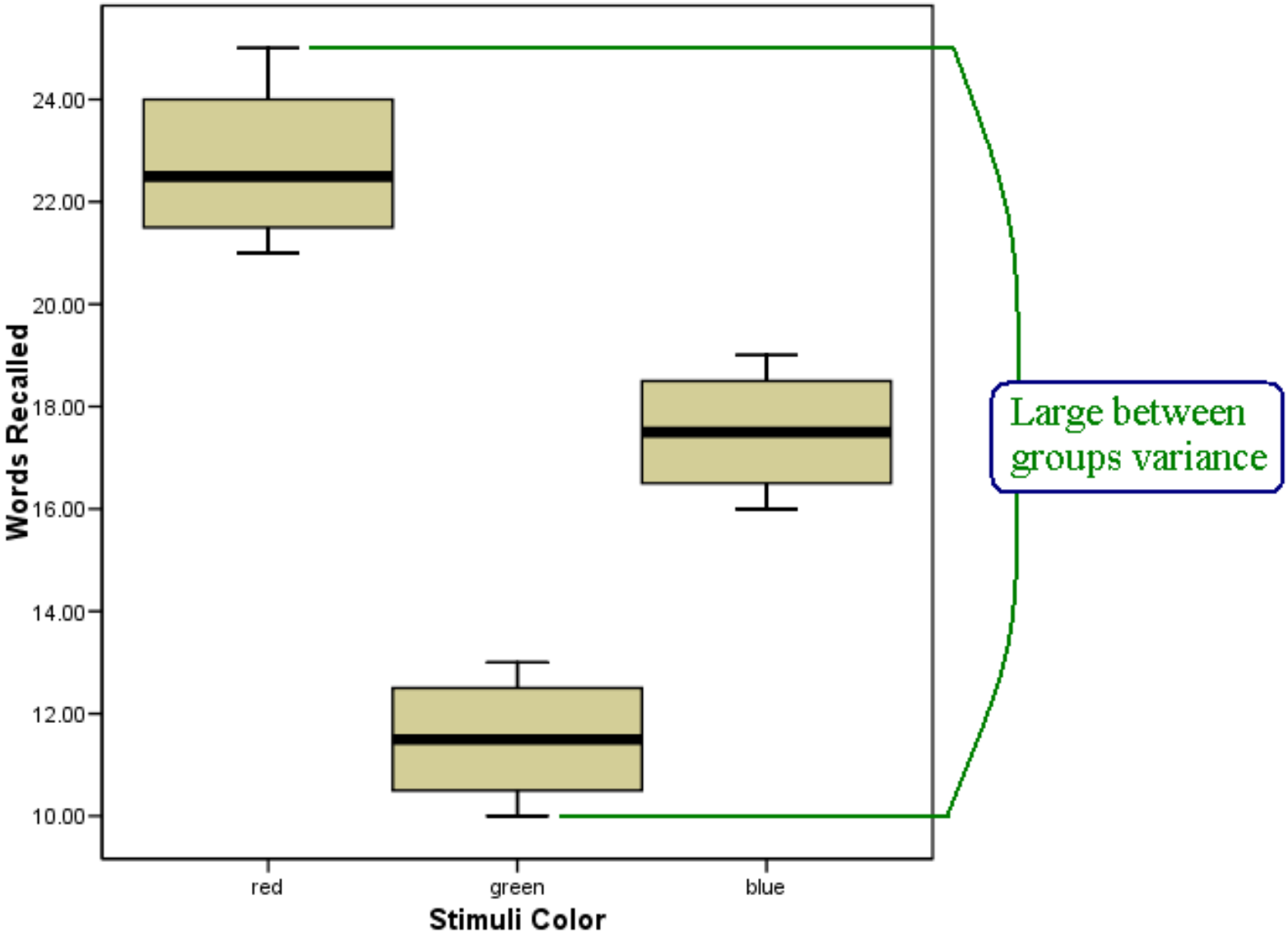
$$F = \frac{MS_b}{MS_w}$$

Box plots show between groups variance



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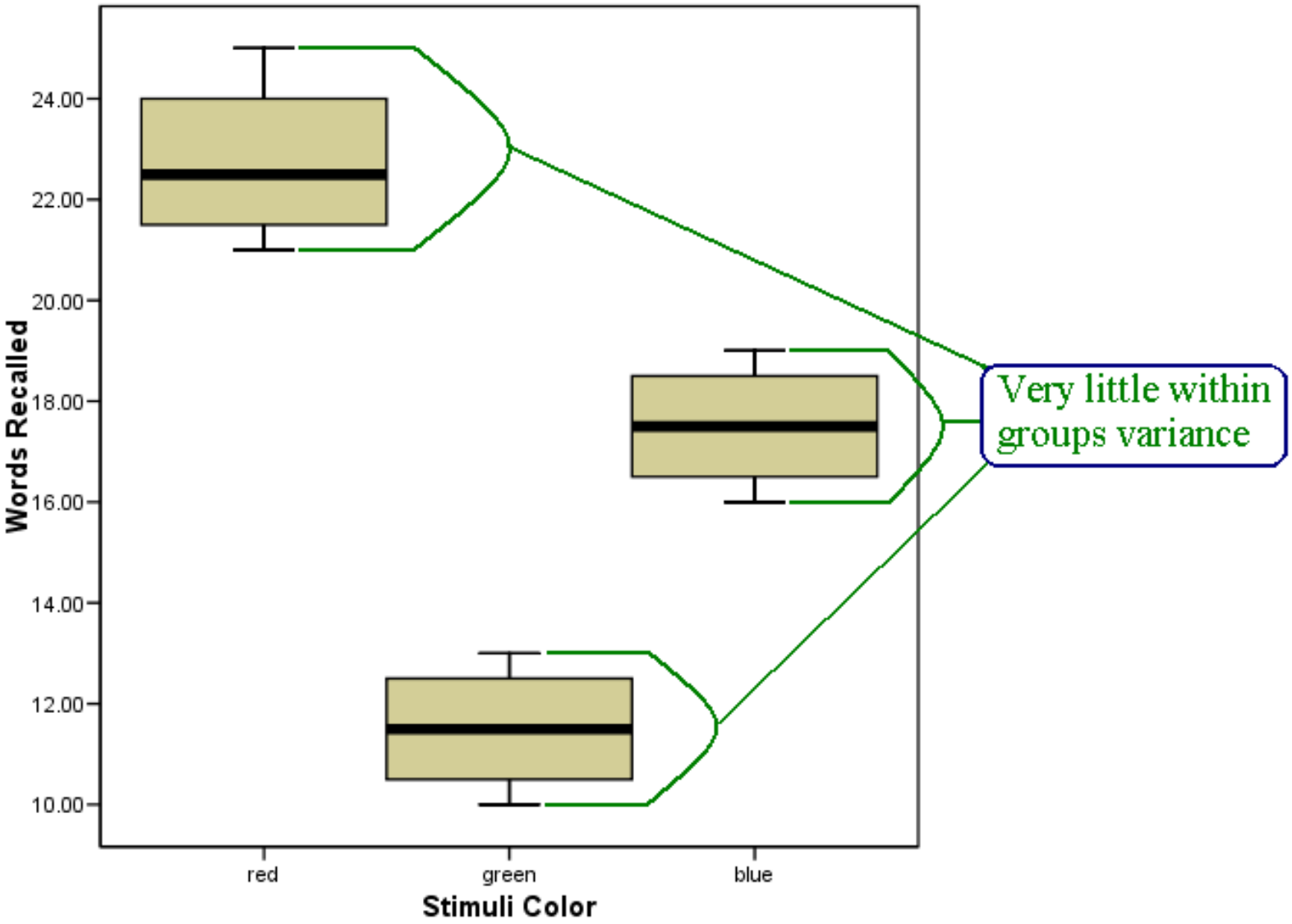

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- Also at the G-power site, a Literature section is available which contains a references list related to power and sample size calculation.

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- The Welch Robust Procedure.
  - *Good News*: Gain in power and protection against Type I error.
  - *Bad News*: Requires using different (somewhat cumbersome) calculations; although most of the symbols you already know from previous usage.

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Welch Formulas

- First, calculate a few new terms w_k and $\overline{X}'_{.}$ where w_k refers to **weight for each group** and $\overline{X}'_{.}$ refers to **weighted grand mean**.

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- For example, the Red group's weight would be the number of individuals in the Red group divided by the variance of the Red group:

$$w_{red} = \frac{n_{red}}{S_{red}^2} = \frac{4}{2.92} = 1.37$$

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Welch Robust Procedure continued

- Next, calculate the F or rather F'' as it is the correct symbol under Welch's procedure.

Welch Robust Procedure continued

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$$F'' = \frac{\frac{\sum w_k \left(\overline{X}_k - \overline{X}'_{.}\right)^2}{k-1}}{1 + \left[\frac{2(k-2)}{k^2-1}\right] * \left[\sum \left(\frac{1}{n_k - 1}\right)\left(1 - \frac{w_k}{\sum w_k}\right)^2\right]}$$

Welch Robust Procedure continued

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$$df' = \frac{k^2-1}{3\sum\left(\frac{1}{n_k-1}\right)\left(1-\frac{w_k}{\sum w_k}\right)^2}$$

Welch Robust Procedure continued

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- Now use df' and $k-1 = df_b$ to find the critical value in the standard F table using df_b as the numerator and df' as the denominator.

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- See the Supplemental Handout for a complete example of the Welch Robust Procedure.

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A firm understanding of the topics covered here and previously will be necessary for understanding future topics.

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This concludes Module 9

Next time Module 10.

- Next time we'll begin covering Regression.
- Until next time; have a nice day.

These slides initially created on: October 15, 2010
These slides last updated on: October 21, 2010

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[1] This document was created in LaTeX using the Beamer package