Module 9: One-Way Analysis of Variance (ANOVA)

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Research and Statistical Support

Introduction to Statistics for the Social Sciences

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In future modules, we will discuss other analysis of the ANOVA family, such as repeated measures and factorial ANOVA, which features more than one independent variable.
1 IV (with > 2 groups) & 1 interval/ratio DV.
Practical Applicability

- 1 IV (with > 2 groups) & 1 interval/ratio DV.
- The benefit: Can test not only a treatment vs. a control group, but more than one treatment and a control group.

Some Examples:
- Comparing taste satisfaction ratings of Coke, Pepsi, Mountain Dew, and Dr. Pepper.
- Comparing homophobia scores of persons who are Atheist, Christian, Jewish, or Muslim.
- Comparing the stimulant effects (in terms of alertness) of Cocaine, Crystal Meth., Coffee, Red Bull, Mountain Dew, and Placebo.
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Assumptions

As with the Independent Samples $t$ Test, if our assumptions are not met, we may have difficulty applying our results to the populations of interest or, we may not have any faith in the actual result of the test (i.e. invalid).

Primary assumptions of one-way ANOVA are exactly as they were for Independent Samples $t$ Test.

- Normality of the population distributions (i.e. each population should be normally distributed).
- Homogeneity of Variance of the population distributions (i.e. each population’s variance should be similar).
- Independence of Observations; meaning, each score should be independent of the other scores in its population (i.e. no twins in the same group).

As with the $t$ test situation, we evaluate these assumptions (of the populations) using the samples as representatives.
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For example, we have a score: $X_{23}^{} = 17$ which indicates this score is located at the second row and the third column; or we could say that 17 is the second score of the third variable because, data are generally arranged in a matrix with the variables as column headings and rows representing cases (i.e. participants).
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- We analyze the variance to test if there is a significant difference (somewhere) among the means.
- Stated another way, we seek to discover mean differences by analyzing the variances.

Some authors (textbooks) use ‘treatment’ instead of ‘between’ and ‘error’ instead of ‘within’. 
Recall that conceptually, variance is the sums of squared deviations (SOS) divided by the degrees of freedom (df):

\[
S^2 = \frac{\sum(X - \bar{X})^2}{n - 1} = \frac{SOS}{df}
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The ratio of between groups variance to within group variance is our $F$ statistic ($F_{calc}$).
The between groups variance: \( MS_b = \frac{SOS_b}{df_b} \)
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Then compute the grand mean: Sum of the means, divided by the number of means: $\bar{X}_s = \sum \bar{X}_j / k$, where $k$ is the number of means (i.e., groups).
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Then, $SOS_b = \sum \left[ n_j \left( \bar{X}_j - \bar{X}_. \right)^2 \right]$ where $n_j$ is the number of scores in the $j$th group.
The between groups variance: \( MS_b = \frac{SOS_b}{df_b} \)

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The sum of, for each group; the number of scores in the group times the group mean minus the grand mean squared.
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\( df_b = \) number of groups minus 1.
\( df_b = k - 1 \)
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Then of course; \( MS_w = \frac{SOS_w}{df_w} \)
Calculate $F$

Simply divide: $MS_b / MS_w = F_{calc}$
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- Simply divide: $MS_b / MS_w = F_{calc}$
- Use the $df_b$ and $df_w$ to look up the critical score on the $F$ distribution table for the significance level you are using (typically 0.05).
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- Simply divide: $MS_b/MS_w = F_{calc}$
- Use the $df_b$ and $df_w$ to look up the critical score on the $F$ distribution table for the significance level you are using (typically 0.05).
- The ‘numerator degrees of freedom’ is $df_b$ while the ‘denominator degrees of freedom’ is $df_w$. 
$SOS_t$ is the sum of the squared deviations of each individual score around the grand mean.
**Total SOS and \( df \)**

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\[
SOS_t = \sum \left( X - \bar{X}_{..} \right)^2
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- It is not necessary to calculate the totals, but often when hand calculating they are useful for checking previous calculations since;

\[ SOS_t = SOS_b + SOS_w \]
\[ df_t = df_b + df_w \]
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Table 1: Example ANOVA Summary Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SOS</th>
<th>df</th>
<th>MS</th>
<th>$F_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>64</td>
<td>2</td>
<td>32</td>
<td>11.59</td>
</tr>
<tr>
<td>Within</td>
<td>58</td>
<td>21</td>
<td>2.76</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>122</td>
<td>23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Between and Within should add to Total, if not there was an error in computation.
Recall, $SOS/df = MS$ and $MS_b/MS_w = F_{calc}$
Another way to *think* about ANOVA

Previously we said: The ratio of between groups variance to within groups variance is our $F$ statistic.

$$F = \frac{MS_b}{MS_w}$$
Another way to *think* about ANOVA

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  \[ F = \frac{MS_d}{MS_w} \]

- This could be thought of as: The ratio of differences based on treatment *and chance*, to the differences based on chance alone, or as the ratio of treatment and error to just error.
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  $$ F = \frac{\text{Treatment Effects} + \text{Error}}{\text{Error}} $$
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- Simple math principles leave just the treatment effects by ‘crossing-out’ the error terms.
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$$F = \frac{\text{Treatment Effects} + \text{Error}}{\text{Error}}$$

Simple math principles leave just the treatment effects by ‘crossing-out’ the error terms.

$$F = \frac{\text{Treatment differences} + \text{Individual differences}}{\text{Individual differences}}$$
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Logic behind NHST One-way ANOVA

• If $H_0$ is true, then all the groups will be very similar; their means will be close to one another.
  • Between groups variance ($MS_b$) will be small.
  • If between groups variance is small, then $F$ will be small as well; and likely not significant ($p < .05$).
• If $H_0$ is not true, then all the groups will be very different.
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If $H_0$ is not true, then all the groups will be very different.
- Between groups variance and $F$ will be larger; and likely significant ($p < .05$).

Conversely; if within groups variance is small, then $F$ will be large and if within groups variance is large, $F$ will be small.

Remember; $F = \frac{MS_b}{MS_w}$
Study of the effects of font color on recall. We were interested in determining which color font contributes to the best recall of words presented.
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Participants were then instructed to watch a presentation of 30 words while trying to memorize each word, each presented for 45 seconds.
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Participants were then instructed to recall as many of the words as possible.
Raw data

- Three colors are used to present words.

Participants are told to memorize the words. There are 30 words, each presented for 45 seconds. Look carefully at the scores. What do you notice?
Raw data

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Three colors are used to present words.

Participants are told to memorize the words.

There are 30 words, each presented for 45 seconds.

Look carefully at the scores.

What do you notice?

<table>
<thead>
<tr>
<th>Font Color</th>
<th>Words Recalled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>25</td>
</tr>
<tr>
<td>Red</td>
<td>22</td>
</tr>
<tr>
<td>Red</td>
<td>23</td>
</tr>
<tr>
<td>Red</td>
<td>21</td>
</tr>
<tr>
<td>Green</td>
<td>10</td>
</tr>
<tr>
<td>Green</td>
<td>12</td>
</tr>
<tr>
<td>Green</td>
<td>11</td>
</tr>
<tr>
<td>Green</td>
<td>13</td>
</tr>
<tr>
<td>Blue</td>
<td>18</td>
</tr>
<tr>
<td>Blue</td>
<td>17</td>
</tr>
<tr>
<td>Blue</td>
<td>19</td>
</tr>
<tr>
<td>Blue</td>
<td>16</td>
</tr>
</tbody>
</table>
### Data and Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Red Group</th>
<th>Green Group</th>
<th>Blue Group</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>10</td>
<td>18</td>
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<td>70</td>
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<td>4</td>
<td>4</td>
<td>4</td>
<td>12</td>
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<td>X_...</td>
<td>22.75</td>
<td>11.5</td>
<td>17.5</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>1.71</td>
<td>1.29</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>S^2</td>
<td>2.92</td>
<td>1.66</td>
<td>1.66</td>
<td>6.24</td>
</tr>
<tr>
<td>df</td>
<td>3</td>
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<tr>
<td>df_t</td>
<td></td>
<td></td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

Research and Statistical Support
NHST Step 1

- Identify the populations and restate the research question as $H_0$ and $H_1$

Null Hypothesis: $H_0: \mu_1 = \mu_2 = \mu_3$

Alternative Hypothesis: $H_1: \mu_1 \neq \mu_2 \neq \mu_3$

Note: The Null Hypothesis could be rejected under several conditions.
NHST Step 1

- Identify the populations and restate the research question as $H_0$ and $H_1$
- Population 1: UNT undergraduates who are presented words with red font.
NHST Step 1

- Identify the populations and restate the research question as $H_0$ and $H_1$
- Population 1: UNT undergraduates who are presented words with red font.
- Population 2: UNT undergraduates who are presented words with green font.

Null Hypothesis: $H_0: \mu_1 = \mu_2 = \mu_3$

Alternative Hypothesis: $H_1: \mu_1 \neq \mu_2 \neq \mu_3$

Note: The Null Hypothesis could be rejected under several conditions.
NHST Step 1

- Identify the populations and restate the research question as $H_0$ and $H_1$
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- Null Hypothesis: $H_0: \mu_1 = \mu_2 = \mu_3$
- Alternative Hypothesis: $H_1: \mu_1 \neq \mu_2 \neq \mu_3$
  - Note: The Null Hypothesis could be rejected under several conditions.
  - Any significant difference among the three means.
NHST Step 2

- Determine the characteristics of the comparison distribution.
NHST Step 2

- Determine the characteristics of the comparison distribution.
- The comparison distribution is the $F$ distribution with 2 and 9 degrees of freedom:
NHST Step 2

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  - $df_b = k - 1 = 3 - 1 = 2$
NHST Step 2

- Determine the characteristics of the comparison distribution.
- The comparison distribution is the $F$ distribution with 2 and 9 degrees of freedom:
  - $df_b = k - 1 = 3 - 1 = 2$
  - $df_w = n_t - k = 12 - 3 = 9$
NHST Step 3

- Determine the cutoff sample score on the comparison distribution at which $H_0$ should be rejected.
NHST Step 3

- Determine the cutoff sample score on the comparison distribution at which $H_0$ should be rejected.
- Using the $F$ distribution with a significance level of 0.05; and 2 (numerator) and 9 (denominator) degrees of freedom: $F_{\text{crit}} = 4.26$
NHST Step 3

- Determine the cutoff sample score on the comparison distribution at which $H_0$ should be rejected.

- Using the $F$ distribution with a significance level of 0.05; and 2 (numerator) and 9 (denominator) degrees of freedom: $F_{crit} = 4.26$

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NHST Step 4

- Determine your sample’s score on the comparison distribution.
NHST Step 4

- Determine your sample’s score on the comparison distribution.
  - Compute your sample statistic(s).
NHST Step 4

- Determine your sample’s score on the comparison distribution.
  - Compute your sample statistic(s).
- Calculate $SOS$, $df$, $MS$, and $F_{calc}$ for Between groups, Within groups, and Total (as a check of the other calculations) given the information provided above.
Calculate $SOS_b$ and $df_b$

Sum of the $n$ of each group times the squared deviations of each group mean from the grand mean.

$$SOS_b = \sum \left[ n_j \left( \bar{X}_j - \bar{X}_{..} \right)^2 \right]$$
Calculate $SOS_b$ and $df_b$

Sum of the $n$ of each group times the squared deviations of each group mean from the grand mean.

$$SOS_b = \sum n_j \left( \bar{X}_j - \bar{X}_{..} \right)^2$$

$$SOS_b = n_1 \left( \bar{X}_1 - \bar{X}_{..} \right)^2 + n_2 \left( \bar{X}_2 - \bar{X}_{..} \right)^2 + n_3 \left( \bar{X}_3 - \bar{X}_{..} \right)^2$$
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$$SOS_b = 4 \left( 22.75 - 17.25 \right)^2 + 4 \left( 11.5 - 17.25 \right)^2 + 4 \left( 17.5 - 17.25 \right)^2 = 253.50$$
Calculate $SOS_b$ and $df_b$

Sum of the $n$ of each group times the squared deviations of each group mean from the grand mean.

$$SOS_b = \sum \left[ n_j \left( \bar{X}_j - \bar{X}_{..} \right)^2 \right]$$

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$$SOS_b = 4 \left( 22.75 - 17.25 \right)^2 + 4 \left( 11.5 - 17.25 \right)^2 + 4 \left( 17.5 - 17.25 \right)^2 = 253.50$$

$$df_b = k - 1 = 3 - 1 = 2$$
Calculate $MS_b$

Calculation of Mean-Square between, an estimate of the between groups variance:
Calculate $MS_b$

Calculation of Mean-Square between, an estimate of the between groups variance:

$$MS_b = \frac{SOS_b}{df_b}$$

$$MS_b = \frac{253.50}{2}$$

$$MS_b = 126.75$$
Calculate $SOS_w$ and $df_w$

The sum of the squared deviations of each group’s score from its group mean.

$$SOS_w = \sum (X_{ij} - \bar{X}_j)^2$$
Calculate \( \text{SOS}_w \) and \( df_w \)

The sum of the squared deviations of each group’s score from its group mean.

\[
\text{SOS}_w = \sum (X_{ij} - \bar{X}_j)^2
\]

\[
\text{SOS}_w = (X_{11} - \bar{X}_1)^2 + (X_{21} - \bar{X}_1)^2 + (X_{31} - \bar{X}_1)^2 +
(X_{41} - \bar{X}_1)^2 + (X_{12} - \bar{X}_2)^2 + ... + (X_{43} - \bar{X}_3)^2
\]
Calculate $SOS_w$ and $df_w$

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$$SOS_w = \sum (X_{ij} - \bar{X}_j)^2$$

$$SOS_w = (X_{11} - \bar{X}_1)^2 + (X_{21} - \bar{X}_1)^2 + (X_{31} - \bar{X}_1)^2 + (X_{41} - \bar{X}_1)^2 + (X_{12} - \bar{X}_2)^2 + \ldots + (X_{43} - \bar{X}_3)^2$$

$$SOS_w = (25 - 22.75)^2 + (22 - 22.75)^2 + (23 - 22.75)^2 + (21 - 22.75)^2 + (10 - 11.5)^2 + \ldots + (16 - 17.5)^2 = 18.75$$

$$df_w = n_t - k = 12 - 3 = 9$$
Calculate $SOS_w$ and $df_w$

The sum of the squared deviations of each group’s score from its group mean.

$$SOS_w = \sum (X_{ij} - \bar{X}_j)^2$$

$$SOS_w = (X_{11} - \bar{X}_1)^2 + (X_{21} - \bar{X}_1)^2 + (X_{31} - \bar{X}_1)^2 + (X_{41} - \bar{X}_1)^2 + (X_{12} - \bar{X}_2)^2 + \ldots + (X_{43} - \bar{X}_3)^2$$

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$$df_w = n_t - k = 12 - 3 = 9$$
Calculate $\text{MS}_w$

Calculation of Mean-Square within (often called Mean-Square error), an estimate of the within groups variance:
Calculation of Mean-Square within (often called Mean-Square error), an estimate of the within groups variance:

\[ MS_w = \frac{SOS_w}{df_w} \]

\[ MS_w = \frac{18.75}{9} \]

\[ MS_w = 2.083 \]
Calculate $SOS_t$ and $df_t$

The sum of the squared deviations of each score from the grand mean.

$$SOS_t = \sum (X_{ij} - \bar{X}_.)^2$$
Calculate $SOS_t$ and $df_t$

The sum of the squared deviations of each score from the grand mean.

$$SOS_t = \sum \left( X_{ij} - \overline{X}_{..} \right)^2$$

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$$SOS_t = (25 - 17.25)^2 + (22 - 17.25)^2 + (23 - 17.25)^2 + (21 - 17.25)^2 + (10 - 17.25)^2 + ... (16 - 17.25)^2 = 272.25$$
Calculate $\text{SOS}_t$ and $df_t$

The sum of the squared deviations of each score from the grand mean.

$$\text{SOS}_t = \sum \left( X_{ij} - \overline{X}.. \right)^2$$

$$\text{SOS}_t = \left( X_{11} - \overline{X}.. \right)^2 + \left( X_{21} - \overline{X}.. \right)^2 + \left( X_{31} - \overline{X}.. \right)^2 + \left( X_{41} - \overline{X}.. \right)^2 + \left( X_{12} - \overline{X}.. \right)^2 + \ldots + \left( X_{43} - \overline{X}.. \right)^2$$

$$\text{SOS}_t = (25 - 17.25)^2 + (22 - 17.25)^2 + (23 - 17.25)^2 + (21 - 17.25)^2 + (10 - 17.25)^2 + \ldots + (16 - 17.25)^2 = 272.25$$

$$df_t = n_t - 1 = 12 - 1 = 11$$
Build the ANOVA Summary Table and Calculate $F$

Table 2: NHST Example ANOVA Summary Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SOS</th>
<th>df</th>
<th>MS</th>
<th>$F_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>253.50</td>
<td>2</td>
<td>126.75</td>
<td>60.84</td>
</tr>
<tr>
<td>Within</td>
<td>18.75</td>
<td>9</td>
<td>2.083</td>
<td></td>
</tr>
<tr>
<td>Total</td>
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<td></td>
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</table>

Recall, $\frac{SOS}{df} = MS$ and $\frac{MS_{b}}{MS_{w}} = F_{calc}$

$F_{calc} = \frac{MS_{b}}{MS_{w}} = \frac{126.75}{2.083} = 60.84$.

Notice that $SOS_{b} + SOS_{w} = SOS_{t}$ and $df_{b} + df_{w} = df_{t}$.
Build the ANOVA Summary Table and Calculate $F$

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Recall, $SOS/df = MS$ and $MS_b/MS_w = F_{calc}$

$$F_{calc} = \frac{MS_b}{MS_w} = \frac{126.75}{2.083} = 60.84$$

Notice that $SOS_b + SOS_w = SOS_t$ and $df_b + df_w = df_t$
Compare and make a decision.

- Decide whether to reject $H_0$

Because $F_{calc} = 60.84 > 4.26 = F_{crit}$

We would reject the null hypothesis and conclude that there was a significant difference among the sample means. This study suggests that there is a difference among the three theoretical populations (each represented by a sample here).

However, we do not yet know where the significant difference is located, and there may be more than one significant difference among the three groups.
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Two possible ways of identifying where the significant difference(-s) lie.
What’s next?

- The one-way ANOVA only tells us if there was some significant difference among the groups.
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- Planned Comparisons, which are *planned* prior to data collection (when designing the study).
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Two possible ways of identifying where the significant difference(-s) lie.

Planned Comparisons, which are *planned* prior to data collection (when designing the study).

- Used to assess specific hypotheses about specific groups (e.g., Red > Blue; Green < Blue, etc.).
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Two possible ways of identifying where the significant difference(-s) lie.

Planned Comparisons, which are *planned* prior to data collection (when designing the study).

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Post-hoc testing, which typically compares all possible pair-wise comparisons of the groups.

- Traditionally, only done when the ‘omnibus’ $F$ is significant.
- Reality; almost always done to explore pair-wise differences.
Also called: a-priori comparisons or planned contrasts.
Planned Comparisons

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- Used to test specific group differences to evaluate specific hypotheses (stated prior to data collection).
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    - Each comparison is used to compare two groups.
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Two general types of planned comparisons.

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  - With only two groups $df_b = 1$ always, because $df_b = k - 1$
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  - Complex Comparisons.
    - Each comparison is used to compare two or more groups.
    - e.g., Orthogonal contrasts.
Suppose, we hypothesized (prior to data collection) that the Red group would recall more words than the Blue group and the Green group would recall less words than the Blue group.
Simple Comparisons

- Suppose, we hypothesized (prior to data collection) that the Red group would recall more words than the Blue group and the Green group would recall less words than the Blue group.
- To do any simple comparison, we need to calculate $F$ for that comparison.
Simple Comparisons

Suppose, we hypothesized (prior to data collection) that the Red group would recall more words than the Blue group and the Green group would recall less words than the Blue group.

To do any simple comparison, we need to calculate $F$ for that comparison.

We need a new $df_b$ (always = 1) and a new grand mean ($\bar{X}_{..}$) for each comparison.
**Simple Comparisons**

- Suppose, we hypothesized (prior to data collection) that the Red group would recall more words than the Blue group and the Green group would recall less words than the Blue group.
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- But, we use the same $SOS_w$ and $df_w$ which leads to the same $MS_w = 2.083$
Simple Comparisons

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Then, for each comparison the usual steps are involved: $F = MS_b/MS_w$
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But, we use the same $SOS_w$ and $df_w$ which leads to the same $MS_w = 2.083$

Then, for each comparison the usual steps are involved: $F = MS_b/MS_w$

You may wish to control for family-wise error rate inflation by dividing the significance level by the number of comparisons.
Recall, we hypothesized (prior to data collection) the Red group would recall more words than the Blue group and the Green group would recall less words than the Blue group. This necessitates two planned comparisons. One to evaluate $H_{01}: \mu_{\text{red}} = \mu_{\text{blue}}$ and $H_{11}: \mu_{\text{red}} > \mu_{\text{blue}}$. One to evaluate $H_{02}: \mu_{\text{green}} = \mu_{\text{blue}}$ and $H_{12}: \mu_{\text{green}} < \mu_{\text{blue}}$. Using $df_{b} = 1$ and $df_{w} = 9$ and with a significance level of 0.05, we get $F_{\text{crit}} = 5.12$ for each comparison. 

http://faculty.vassar.edu/lowry/apx_d.html
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Recall, we hypothesized (prior to data collection) the Red group would recall more words than the Blue group and the Green group would recall less words than the Blue group. This necessitates two planned comparisons.

- One to evaluate $H_{01} : \mu_{\text{red}} = \mu_{\text{blue}}$ and $H_{11} : \mu_{\text{red}} > \mu_{\text{blue}}$
- One to evaluate $H_{02} : \mu_{\text{green}} = \mu_{\text{blue}}$ and $H_{12} : \mu_{\text{green}} < \mu_{\text{blue}}$
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Using $df_b = 1$ and $df_w = 9$ and with a significance level of 0.05, we get $F_{crit} = 5.12$ for each comparison.

http://faculty.vassar.edu/lowry/apx_d.html
Planned Comparison 1: Red ($\bar{X} = 22.75$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_. = 20.125 \quad df_b = k - 1 = 2 - 1 = 1$
Planned Comparison 1: Red ($\bar{X} = 22.75$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_. = 20.125 \quad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left( \bar{X}_j - \bar{X}_. \right)^2$
Planned Comparison 1: Red ($\bar{X} = 22.75$) versus Blue ($\bar{X} = 17.50$)

\[ \bar{X}_\text{..} = 20.125 \quad df_b = k - 1 = 2 - 1 = 1 \]

\[ SOS_b = \sum n_j \left( \bar{X}_j - \bar{X}_\text{..} \right)^2 \]

\[ SOS_b = 4 (22.75 - 20.125)^2 + 4 (17.50 - 20.125)^2 = 55.12 \]
Planned Comparison 1: Red ($\bar{X} = 22.75$) versus Blue ($\bar{X} = 17.50$)

\[ \bar{X}_{..} = 20.125 \quad df_b = k - 1 = 2 - 1 = 1 \]

\[ SOS_b = \sum n_j \left( \bar{X}_j - \bar{X}_{..} \right)^2 \]

\[ SOS_b = 4 (22.75 - 20.125)^2 + 4 (17.50 - 20.125)^2 = 55.12 \]

\[ MS_b = SOS_b / df_b = 55.12 / 1 = 55.12 \]

So, $F_{calc} = 26.46 > 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Red group recalled significantly more words than the Blue group.
Planned Comparison 1: Red ($\bar{X} = 22.75$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_\cdot = 20.125 \quad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left( \bar{X}_j - \bar{X}_\cdot \right)^2$

$SOS_b = 4 (22.75 - 20.125)^2 + 4 (17.50 - 20.125)^2 = 55.12$

$MS_b = SOS_b / df_b = 55.12 / 1 = 55.12$

$SOS_w$ and $df_w$ are the same as was used for the omnibus $F$, so $MS_w$ is the same as well: 2.083
Planned Comparison 1: Red ($\bar{X} = 22.75$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_2 = 20.125 \quad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j (\bar{X}_j - \bar{X}_..)^2$

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$MS_b = SOS_b / df_b = 55.12 / 1 = 55.12$

$SOS_w$ and $df_w$ are the same as was used for the omnibus $F$, so $MS_w$ is the same as well: 2.083

$F_{calc} = MS_b / MS_w = 55.12 / 2.083 = 26.46$

$F_{calc} > 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Red group recalled significantly more words than the Blue group.
Planned Comparison 1: Red ($\bar{X} = 22.75$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_. = 20.125 \quad df_b = k - 1 = 2 - 1 = 1$

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$F_{calc} = MS_b / MS_w = 55.12 / 2.083 = 26.46$

So; $F_{calc} = 26.46 \geq 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Red group recalled significantly more words than the Blue group.
Planned Comparison 2: Green ($\bar{X} = 11.50$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_w = 14.50 \quad df_b = k - 1 = 2 - 1 = 1$
Planned Comparison 2: Green ($\bar{X} = 11.50$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_. = 14.50$  \hspace{1cm} df$_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j (X_j - \bar{X}_.)^2$

$F_{calc} = \frac{MS_b}{MS_w} = \frac{72.00}{2.083} = 34.57$

So, $F_{calc} = 34.57 > 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Green group recalled significantly fewer words than the Blue group.
Planned Comparison 2: Green ($\bar{X} = 11.50$) versus Blue ($\bar{X} = 17.50$)

\[
\bar{X}_\cdot = 14.50 \quad df_b = k - 1 = 2 - 1 = 1
\]

\[
SOS_b = \sum n_j (X_j - \bar{X}_\cdot)^2
\]

\[
SOS_b = 4 (11.50 - 14.50)^2 + 4 (11.50 - 14.50)^2 = 72.0
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So, $F_{calc} = 34.57 > 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Green group recalled significantly fewer words than the Blue group.
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$\bar{X}_. = 14.50 \quad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j \left( \bar{X}_j - \bar{X}_. \right)^2$

$SOS_b = 4 (11.50 - 14.50)^2 + 4 (11.50 - 14.50)^2 = 72.0$

$MS_b = SOS_b / df_b = 72.00 / 1 = 72.00$
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$\bar{X}_- = 14.50 \quad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j (\bar{X}_j - \bar{X}_-)^2$

$SOS_b = 4 (11.50 - 14.50)^2 + 4 (11.50 - 14.50)^2 = 72.0$

$MS_b = SOS_b / df_b = 72.00 / 1 = 72.00$

$SOS_w$ and $df_w$ are the same as was used for the omnibus $F$, so $MS_w$ is the same as well: 2.083

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So; $F_{calc} = 34.57 > 5.12 = F_{crit}$ we reject the null hypothesis and conclude that the Green group recalled significantly fewer words than the Blue group.
Planned Comparison 2: Green ($\bar{X} = 11.50$) versus Blue ($\bar{X} = 17.50$)

$\bar{X}_n = 14.50 \quad df_b = k - 1 = 2 - 1 = 1$

$SOS_b = \sum n_j (\bar{X}_j - \bar{X}_n)^2$

$SOS_b = 4 (11.50 - 14.50)^2 + 4 (11.50 - 14.50)^2 = 72.0$

$MS_b = SOS_b / df_b = 72.00 / 1 = 72.00$

$SOS_w$ and $df_w$ are the same as was used for the omnibus $F$, so $MS_w$ is the same as well: $2.083$

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Sometimes called the Bonferroni Procedure (after it’s founder), it is used to control family-wise error inflation when multiple tests are done in one study.
Family-wise and Pair-wise Error Rates

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Often necessary to find an $F$ distribution table with exact values, this can be difficult because most tables are abbreviated for common cutoff points. Should be considered when doing planned comparisons. Some post-hoc tests have embedded controls for $FW$ error rates.
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For example, if doing 3 comparisons and wanting a total significance level of 0.05, then:

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In exploratory research, you would test all possible combinations of groups to determine where the significant differences are located (typically only if the omnibus $F$ is significant).
Post-hoc Testing

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Most researchers always do either planned comparisons (preferred) or post-hoc testing (exploratory), because the omnibus $F$ is not terribly informative on its own.
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  - If unequal group sizes: Games-Howell.
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More on Post-hoc tests in general

- Most post-hoc tests rely on the $t$ distribution or a modification of it.

FYI: $F = t^2$, which means the simple (two group) comparisons above can be converted with $t = \sqrt{F}$.

The modified $t$ distribution is often called the Studentized $t$ statistic. Symbol for the Studentized $t$ is $q$.

It is defined by:

$$q = \frac{X_l - X_s}{\sqrt{MS_w}}$$

where $X_l$ refers to the largest mean among the groups, $X_s$ the smallest, and $n_g$ is the number of individuals per group.

[http://www.stat.duke.edu/courses/Spring98/sta110c/qtable.html](http://www.stat.duke.edu/courses/Spring98/sta110c/qtable.html)
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More on $q$

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- There are 3 means in this set, $r = 3$ which is used in looking up values in the $q$ table (the table below uses the familiar $k$ instead of $r$ but, most use $r$).
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  q_{\text{calc}} = \frac{X_l - X_s}{\sqrt{MS_w}} = \frac{22.75 - 11.50}{\sqrt{2.083}} = 15.58
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  \( q_{\text{calc}} = 15.58 \) and we can use \( r = 3 \), significance level of 0.05, and \( df = 9 \) to find \( q_{\text{crit}} = 3.95 \) in the \( q \) table.

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Which can replace the omnibus \( F \), but with less statistical power.
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- Using the formula from above, we can solve for $q$

$$q = \frac{X_l - X_s}{\sqrt{MS_w}} = \frac{22.75 - 11.50}{\sqrt{2.083^4}} = 15.58$$

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Tukey’s Honestly Significant Differences test

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\bar{X}_i - \bar{X}_j = q_{crit} \sqrt{\frac{MS_w}{n_g}} = 3.95 \sqrt{\frac{2.083}{4}} = 2.85
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\bar{X}_i - \bar{X}_j = q_{crit} \sqrt{\frac{MS_w}{n_g}} = 3.95 \sqrt{\frac{2.083}{4}} = 2.85
\]

- So, if a mean difference \( (\bar{X}_i - \bar{X}_j) \) is larger than 2.85 we would conclude there is a significant difference between those two means.
Tukey’s HSD continued

With our minimum significant difference calculated at 2.85; we can compare the differences among our three means to it, to determine which means differ significantly from the others.
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<table>
<thead>
<tr>
<th></th>
<th>Green</th>
<th>Blue</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{r,.05} = 2.85$</td>
<td>11.50</td>
<td>17.50</td>
<td>22.75</td>
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<tr>
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<td>6.00</td>
<td>11.25</td>
</tr>
<tr>
<td>Blue = 17.50</td>
<td>–</td>
<td>0</td>
<td>5.25</td>
</tr>
<tr>
<td>Red = 22.75</td>
<td>–</td>
<td>–</td>
<td>0</td>
</tr>
</tbody>
</table>
Tukey’s HSD continued

With our minimum significant difference calculated at 2.85; we can compare the differences among our three means to it, to determine which means differ *significantly* from the others.

\[
q_{r,.05} = 2.85
\]

<table>
<thead>
<tr>
<th></th>
<th>Green</th>
<th>Blue</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green = 11.50</td>
<td>0</td>
<td>6.00</td>
<td>11.25</td>
</tr>
<tr>
<td>Blue = 17.50</td>
<td>–</td>
<td>0</td>
<td>5.25</td>
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<tr>
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<td>–</td>
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<td>0</td>
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So, we find a significant difference between each pair of means because, each difference is greater than 2.85.
The Tukey test above assumes equal sample sizes for each group and equal variances among the groups.
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Essentially, this test incorporates the samples sizes and variances of each group being compared.
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The $df$ and the critical difference between means ($\bar{X}_i - \bar{X}_j$) are modified for inclusion of samples sizes and variances.
Until now we have consistently used $df_w$ for finding our critical values (e.g., $q_{crit}$).
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With the Games-Howell test, we actually calculate $d'_{f}$ using each groups’ sample size and variance; such that for each pair of groups:
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With the Games-Howell test, we actually calculate $df'$ using each groups’ sample size and variance; such that for each pair of groups:

$$df' = \frac{\left(\frac{s_i^2}{n_i} + \frac{s_j^2}{n_j}\right)^2}{\left(\frac{s_i^2}{n_i}\right)^2 + \left(\frac{s_j^2}{n_j}\right)^2}$$
Games-Howell modification of $df$

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- where subscript $i$ and subscript $j$ identify descriptive statistics from each group being compared.
Example $df'$

As an example let’s consider the Red and Green groups.

- Red (i): $\bar{X} = 22.75$, $S^2 = 2.92$, $n = 4$
- Green (j): $\bar{X} = 11.50$, $S^2 = 1.66$, $n = 4$
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Pay careful attention to the fact that $S^2$ is a symbol, not an operation.
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- Pay careful attention to the fact that \( S^2 \) is a symbol, not an operation.
  - See the supplemental handout for an example of the complete, step-by-step calculation.
Games-Howell difference between means

Minimum Significant Difference

So, given \( r = 3 \), significance level of 0.05, and now \( df = 5.787 \) we look to the \( q \) table and find: \( q_{\text{crit}} \approx 4.60 \).

http://www.stat.duke.edu/courses/Spring98/sta110c/qtable.html

Where earlier we had:

\[
X_i - X_j = q_{\text{crit}} \sqrt{MS_w} \]

Now, for each pair of means, we have:

\[
X_i - X_j = q_{\text{crit}} \sqrt{S_i^2/n_i + S_j^2/n_j} \]

For the current Red vs. Green example:

\[
X_i - X_j = 5.787 \sqrt{2.924 + 1.664} = 4.379 \]
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  - Therefore, the Games-Howell test is highly recommended.
The ANOVA family of analyses have a variety of effect size measures which can be used.
Effect Size Measures for ANOVA

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- Recall, there are generally two forms of effect size measures;
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GLM will be discussed in more detail in future modules (e.g., regression) but, suffice to say; we have specified a linear model here, which generally refers to a linear relationship between the variables we are analyzing.
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  - Omega squared is virtually unbiased.
Eta squared

- Eta squared ($\eta^2$) is very easy to calculate, but as mentioned above it is biased.

\[ \eta^2 = \frac{SOS_b}{SOS_t} = \frac{253.50}{272.25} = 0.9311 \]

Which suggests 93.11% of the variance in words recalled (Dependent variable [DV]) is attributable to the differences in font color (Independent variable [IV]).
### Eta squared

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<tr>
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<tr>
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$$R^2 = \frac{(F)(df_b)}{(F)(df_b) + df_w} = \frac{60.84(2)}{60.84(2) + 9} = \frac{121.68}{130.68} = .9311$$

No surprise here, 93.11% of the variance in words recalled (DV) is attributable to the differences in font color (IV). This is the same effect size we got for $\eta^2$.
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$$\omega^2 = \frac{SOS_b - (k - 1)MS_w}{SOS_t + MS_w} = \frac{253.5 - (3 - 1)2.083}{272.25 + 2.083} = \frac{249.332}{274.33} = .9089$$

Here we see a less biased, more accurate representation of the relationship between the IV and the DV. 90.89% of the variance in words recalled (DV) is attributable to the differences in font color (IV). Please note; Omega squared should also be calculated on the results of Planned Comparisons if they were performed (examples on next slide).
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Omega squared for planned comparisons

For Planned comparison 1 from earlier:

\[ \omega^2 = \frac{S_{OSS} - (k - 1)MS_{w}}{S_{OSS} + MS_{w}} = 53.037 \]

So, 19.33% of the variance in words recalled is attributable to the difference between Red and Blue font color.

For Planned Comparison 2 from earlier:

\[ \omega^2 = \frac{S_{OSS} - (k - 1)MS_{w}}{S_{OSS} + MS_{w}} = 69.917 \]

So, 25.49% of the variance in words recalled is attributable to the difference between Green and Blue font color.
Omega squared for planned comparisons

For Planned comparison 1 from earlier:

\[ \omega^2 = \frac{SOS_b - (k - 1)MS_w}{SOS_t + MS_w} = \frac{55.12 - (2 - 1)2.083}{272.25 + 2.083} = \frac{53.037}{274.33} = .1933 \]
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So, 19.33% of the variance in words recalled is attributable to the difference between Red and Blue font color.
Omega squared for planned comparisons

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So, 19.33% of the variance in words recalled is attributable to the difference between Red and Blue font color.

- For Planned Comparison 2 from earlier:

\[ \omega^2 = \frac{SOS_b - (k - 1)MS_w}{SOS_t + MS_w} = \frac{72 - (2 - 1)2.083}{272.25 + 2.083} = \frac{69.917}{274.33} = .2549 \]

So, 25.49% of the variance in words recalled is attributable to the difference between Green and Blue font color.
For Planned comparison 1 from earlier:

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- In the situation of testing individual pairs of means, we could review the previous module (*t* tests) and apply those methods to the current testing of individual pairs of means – for calculating a CI (and Cohen’s *d*).
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  - Recall the general equations for CI limits:
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- Or, we could use current information and our general understanding of the equations for the upper limit (UL) and lower limit (LL) of a CI.
  - Recall the general equations for CI limits:

\[
UL = (+\text{crit}) \times (SE) + \text{mean} \\
LL = (-\text{crit}) \times (SE) + \text{mean}
\]
During planned comparison testing, we were interested in the difference between pairs of group means and we had all the elements we needed to compute the UL and LL for each comparison.
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- Here we apply those elements to the general form of the UL and LL equations for **each Planned Comparison**.
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- Here we apply those elements to the general form of the UL and LL equations for each **Planned Comparison**.
  
- However, we are now in a two group comparison (essentially *t* testing) so given that $F = t^2$ we need to take the square root of $F_{crit}$ and $MS_w$. 

---

**CI for Planned Comparisons**

*Research and Statistical Support*
During planned comparison testing, we were interested in the *difference between* pairs of group means and we had all the elements we needed to compute the UL and LL for each comparison.

- Here we apply those elements to the general form of the UL and LL equations for **each Planned Comparison**.

- However, we are now in a two group comparison (essentially $t$ testing) so given that $F = t^2$ we need to take the square root of $F_{crit}$ and $MS_w$.

\[
UL = \sqrt{(F_{crit} + 1) \cdot (MS_w)} + \bar{X}_i - \bar{X}_j \\
LL = \sqrt{(F_{crit} - 1) \cdot (MS_w)} + \bar{X}_i - \bar{X}_j
\]
Adjustments for square root of a negative

\[ UL = \sqrt{(+F_{crit}) \times (MS_w)} + X_i - X_j \]
\[ LL = \sqrt{(-F_{crit}) \times (MS_w)} + X_i - X_j \]
As many of you know, you cannot take the square root of a negative number.
Adjustments for square root of a negative

\[ UL = \sqrt{ (+F_{crit}) \ast (MS_w) + X_i - X_j } \]
\[ LL = \sqrt{ (-F_{crit}) \ast (MS_w) + X_i - X_j } \]

- As many of you know, you can not take the square root of a negative number.
- So, recognize that the formulas at the bottom are equivalent for what we want to do at the top.
Adjustments for square root of a negative

\[
UL = \sqrt{(+F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j \\
LL = \sqrt{(-F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j
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- As many of you know, you can not take the square root of a negative number.
- So, recognize that the formulas at the bottom are equivalent for what we want to do at the top.

\[
UL = \sqrt{(F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j \\
LL = \sqrt{(F_{crit}) \times (MS_w)} - \bar{X}_i - \bar{X}_j
\]
CI for Example Planned Comparisons (PC)

- CI for PC 1: Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)

\[
UL = \sqrt{+F_{\text{crit}} \times (MS_w)} + (X_i - X_j) = \sqrt{5.12 \times 2.083} + 5.25 = 8.52 \\
LL = \sqrt{-F_{\text{crit}} \times (MS_w)} + (X_i - X_j) = \sqrt{5.12 \times 2.083} - 5.25 = 1.98
\]

If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.52 and 1.98.

- CI for PC 2: Green ($\bar{X} = 11.50$) vs. Blue ($\bar{X} = 17.50$)

\[
UL = \sqrt{+F_{\text{crit}} \times (MS_w)} + (X_i - X_j) = \sqrt{5.12 \times 2.083} + 6.00 = 9.27 \\
LL = \sqrt{-F_{\text{crit}} \times (MS_w)} + (X_i - X_j) = \sqrt{5.12 \times 2.083} - 6.00 = 2.73
\]

If we drew an infinite number of samples, we would expect 95% of the mean differences between the Green and Blue groups to be between 9.27 and 2.73.
Cl for Example Planned Comparisons (PC)

Cl for PC 1: Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)

\[
UL = \sqrt{(+F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j = \sqrt{5.12 \times (2.083)} + 5.25 = 8.52
\]

\[
LL = \sqrt{(-F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j = \sqrt{5.12 \times (2.083)} - 5.25 = 1.98
\]

If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.52 and 1.98.

Cl for PC 2: Green ($\bar{X} = 11.50$) vs. Blue ($\bar{X} = 17.50$)

\[
UL = \sqrt{(+F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j = \sqrt{5.12 \times (2.083)} + 6.00 = 9.27
\]

\[
LL = \sqrt{(-F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j = \sqrt{5.12 \times (2.083)} - 6.00 = 2.73
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If we drew an infinite number of samples, we would expect 95% of the mean differences between the Green and Blue groups to be between 9.27 and 2.73.
Cl for Example Planned Comparisons (PC)

- Cl for PC 1: Red ($\overline{X} = 22.75$) vs. Blue ($\overline{X} = 17.50$)

$$UL = \sqrt{( + F_{crit}) \times (MS_w)} + \overline{X}_i - \overline{X}_j = \sqrt{5.12 \times (2.083)} + 5.25 = 8.52$$

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If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.52 and 1.98.
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- CI for PC 1: Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)

\[ UL = \sqrt{(+F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j = \sqrt{5.12 \times 2.083} + 5.25 = 8.52 \]
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If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.52 and 1.98.

- CI for PC 2: Green ($\bar{X} = 11.50$) vs. Blue ($\bar{X} = 17.50$)
CI for Example Planned Comparisons (PC)

- CI for PC 1: Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)

$$UL = \sqrt{(+F_{crit} \times (MS_w)) + \bar{X}_i - \bar{X}_j} = \sqrt{5.12 \times (2.083)} + 5.25 = 8.52$$

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If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.52 and 1.98.

- CI for PC 2: Green ($\bar{X} = 11.50$) vs. Blue ($\bar{X} = 17.50$)

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- CI for PC 1: Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)

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  If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.52 and 1.98.

- CI for PC 2: Green ($\bar{X} = 11.50$) vs. Blue ($\bar{X} = 17.50$)

  \[
  UL = \sqrt{(+F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j = \sqrt{5.12 \times (2.083)} + 6.00 = 9.27
  \]

  \[
  LL = \sqrt{(-F_{crit}) \times (MS_w)} + \bar{X}_i - \bar{X}_j = \sqrt{5.12 \times (2.083)} - 6.00 = 2.73
  \]

  If we drew an infinite number of samples, we would expect 95% of the mean differences between the Green and Blue groups to be between 9.27 and 2.73.
With Tukey’s Post-hoc testing, we are using modified $t$ tests, so we can simply use the calculated **minimum difference between means** (2.85) from earlier.
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- CI for PH 1: Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)
With Tukey’s Post-hoc testing, we are using modified $t$ tests, so we can simply use the calculated \textbf{minimum difference between means} (2.85) from earlier.

- CI for PH 1: Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)

\[ UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 5.25 = 8.10 \]
\[ LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 5.25 = 2.40 \]

If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.10 and 2.40.

Continued on next slide.
With Tukey’s Post-hoc testing, we are using modified \( t \) tests, so we can simply use the calculated **minimum difference between means** (2.85) from earlier.

- **CI for PH 1: Red (\( \bar{X} = 22.75 \)) vs. Blue (\( \bar{X} = 17.50 \))**

  \[
  UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 5.25 = 8.10
  \]
  \[
  LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 5.25 = 2.40
  \]

- If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.10 and 2.40.
Cl for Tukey’s HSD Post-hoc (PH) test

With Tukey’s Post-hoc testing, we are using modified $t$ tests, so we can simply use the calculated **minimum difference between means** (2.85) from earlier.

- **Cl for PH 1:** Red ($\bar{X} = 22.75$) vs. Blue ($\bar{X} = 17.50$)

  $$UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 5.25 = 8.10$$
  $$LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 5.25 = 2.40$$

- If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Blue groups to be between 8.10 and 2.40.
- Continued on next slide.
CI for Tukey’s HSD PH continued

- CI for PH 2: Red ($\bar{X} = 22.75$) vs. Green ($\bar{X} = 11.50$)
CI for Tukey’s HSD PH continued

- CI for PH 2: Red ($\bar{X} = 22.75$) vs. Green ($\bar{X} = 11.50$)

  \[ UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 11.25 = 14.10 \]
  \[ LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 11.25 = 8.40 \]

If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Green groups to be between 14.10 and 8.40.

- CI for PH 3: Blue ($\bar{X} = 17.50$) vs. Green ($\bar{X} = 11.50$)

  \[ UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 6.00 = 8.85 \]
  \[ LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 6.00 = 3.15 \]

If we drew an infinite number of samples, we would expect 95% of the mean differences between the Green and Blue groups to be between 8.85 and 3.15.
CI for Tukey’s HSD PH continued

- CI for PH 2: Red ($\bar{X} = 22.75$) vs. Green ($\bar{X} = 11.50$)
  
  \[
  UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 11.25 = 14.10 \\
  LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 11.25 = 8.40
  \]

- If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Green groups to be between 14.10 and 8.40.
CI for Tukey’s HSD PH continued

- CI for PH 2: Red ($\bar{X} = 22.75$) vs. Green ($\bar{X} = 11.50$)

  \[ UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 11.25 = 14.10 \]
  \[ LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 11.25 = 8.40 \]

- If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Green groups to be between 14.10 and 8.40.

- CI for PH 3: Blue ($\bar{X} = 17.50$) vs. Green ($\bar{X} = 11.50$)
Cl for Tukey's HSD PH continued

- Cl for PH 2: Red ($\bar{X} = 22.75$) vs. Green ($\bar{X} = 11.50$)

  $$UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 11.25 = 14.10$$
  $$LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 11.25 = 8.40$$

  If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Green groups to be between 14.10 and 8.40.

- Cl for PH 3: Blue ($\bar{X} = 17.50$) vs. Green ($\bar{X} = 11.50$)

  $$UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 6.00 = 8.85$$
  $$LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 6.00 = 3.15$$
**Cl for Tukey’s HSD PH continued**

- **Cl for PH 2: Red ($\bar{X} = 22.75$) vs. Green ($\bar{X} = 11.50$)**

  $UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 11.25 = 14.10$

  $LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 11.25 = 8.40$

- If we drew an infinite number of samples, we would expect 95% of the mean differences between the Red and Green groups to be between 14.10 and 8.40.

- **Cl for PH 3: Blue ($\bar{X} = 17.50$) vs. Green ($\bar{X} = 11.50$)**

  $UL = (+q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = +2.85 + 6.00 = 8.85$

  $LL = (-q_{r=3,.05}) + \bar{X}_i - \bar{X}_j = -2.85 + 6.00 = 3.15$

- If we drew an infinite number of samples, we would expect 95% of the mean differences between the Green and Blue groups to be between 8.85 and 3.15.
Simple Default graph from some software

Not very informative.
Simple Default graph from some software

Not very informative.

Displays the means as points.
Simple Default graph from some software

Not very informative.

Displays the means as points

Confusing because the points are linked by a line when we are testing for differences among them.
Simple Default graph from some software

Not very informative.

Displays the means as points

Confusing because the points are linked by a line when we are testing for differences among them.

Also, it shows little of the variance in an Analysis of Variance setting.
Simple Default graph from some software

Not very informative.

Displays the means as points

Confusing because the points are linked by a line when we are testing for differences among them.

Also, it shows little of the variance in an Analysis of Variance setting.
Grouped Box plot represents the data better.

Displays the means as solid black lines in the middle of the boxes.
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Between groups variance is represented by the distance between the top whisker and bottom whisker across groups (i.e. top Red, bottom Green).
Grouped Box plot represents the data better.

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Between groups variance is represented by the distance between the top whisker and bottom whisker across groups (i.e. top Red, bottom Green).

Within groups variance is represented by the distance between the boundary of each box’s top and bottom whiskers.
Grouped Box plot represents the data better.

Displays the means as solid black lines in the middle of the boxes.

Between groups variance is represented by the distance between the top whisker and bottom whisker across groups (i.e. top Red, bottom Green).

Within groups variance is represented by the distance between the boundary of each box’s top and bottom whiskers.
Quick Review

- Recall, $MS_b$ represents the average between groups variance.
Quick Review

- Recall, $MS_b$ represents the average between groups variance.
- And, $MS_w$ represents the average within groups variance.
Quick Review

- Recall, $MS_b$ represents the average between groups variance.
- And, $MS_w$ represents the average within groups variance.
- ANOVA analyzes variance to determine if mean differences are significant.
Quick Review

- Recall, $MS_b$ represents the average between groups variance.
- And, $MS_w$ represents the average within groups variance.
- ANOVA analyzes variance to determine if mean differences are significant.

$$F = \frac{MS_b}{MS_w}$$
Box plots show between groups variance

Large between groups variance
Box plots show within groups variance.
As mentioned in previous modules, post-hoc power (calculated after conducting the analysis) is meaningless.
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- Power is a direct function of sample size.
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- Power is a direct function of sample size.

The best way to use power is to specify it and the effect size you would like to attain, and use those to calculate the number of subjects/participants needed to achieve those goals.
As mentioned in previous modules, post-hoc power (calculated after conducting the analysis) is meaningless.

- Power is a direct function of sample size.

The best way to use power is to specify it and the effect size you would like to attain, and use those to calculate the number of subjects/participants needed to achieve those goals.

- This is called a-priori power calculation.
As mentioned in previous modules, post-hoc power (calculated after conducting the analysis) is meaningless.

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The best way to use power is to specify it and the effect size you would like to attain, and use those to calculate the number of subjects/participants needed to achieve those goals.

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A-priori power calculations (i.e. calculating the necessary sample size) should always be done when planning a study.
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The best way to use power is to specify it and the effect size you would like to attain, and use those to calculate the number of subjects/participants needed to achieve those goals.

- This is called a-priori power calculation.

A-priori power calculations (i.e. calculating the necessary sample size) should always be done when planning a study.

- A-priori calculations should be done prior to data collection.
The best way to calculate a-priori power/sample size is to use G-power 3, which is available for free at the link below.

[http://www.psycho.uni-duesseldorf.de/abteilungen/aap/gpower3/](http://www.psycho.uni-duesseldorf.de/abteilungen/aap/gpower3/)
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At the G-power 3 web site, you will find links to the User Manual for G-power 3 which are very easy to follow and guide you through the process of carrying out the necessary calculations for a variety of analyses.
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Also at the G-power site, a Literature section is available which contains a references list related to power and sample size calculation.
As mentioned in the discussion of the Games-Howell post-hoc test, ANOVA can be biased when data are not evenly distributed among the groups and/or the groups have heterogeneous variances (i.e., violated assumptions).
As mentioned in the discussion of the Games-Howell post-hoc test, ANOVA can be biased when data are not evenly distributed among the groups and/or the groups have heterogeneous variances (i.e., violated assumptions). Fortunately, there exists a very good alternative when data do not fit the standard ANOVA assumptions.
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The Welch Robust Procedure.
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The Welch Robust Procedure.

- **Good News**: Gain in power and protection against Type I error.
As mentioned in the discussion of the Games-Howell post-hoc test, ANOVA can be biased when data are not evenly distributed among the groups and/or the groups have heterogeneous variances (i.e., violated assumptions).

Fortunately, there exists a very good alternative when data do not fit the standard ANOVA assumptions.

The Welch Robust Procedure.

- **Good News**: Gain in power and protection against Type I error.
- **Bad News**: Requires using different (somewhat cumbersome) calculations; although most of the symbols you already know from previous usage.
First, calculate a few new terms $w_k$ and $\bar{X}'$ where $w_k$ refers to weight for each group and $\bar{X}'$ refers to weighted grand mean.
Welch Formulas

- First, calculate a few new terms \( w_k \) and \( \bar{X}' \) where \( w_k \) refers to weight for each group and \( \bar{X}' \) refers to weighted grand mean.

\[
w_k = \frac{n_k}{S_k^2}
\]
First, calculate a few new terms $w_k$ and $\bar{X}'$ where $w_k$ refers to weight for each group and $\bar{X}'$ refers to weighted grand mean.

$$w_k = \frac{n_k}{S^2_k}$$

$$\bar{X}' = \frac{\sum (w_k \bar{X}_k)}{\sum w_k}$$
First, calculate a few new terms $w_k$ and $\overline{X}'$ where $w_k$ refers to weight for each group and $\overline{X}'$ refers to weighted grand mean.

$$w_k = \frac{n_k}{S_k^2}$$

$$\overline{X}' = \frac{\sum (w_k \overline{X}_k)}{\sum w_k}$$

For example, the Red group’s weight would be the number of individuals in the Red group divided by the variance of the Red group: 

$$w_{\text{red}} = \frac{n_{\text{red}}}{S_{\text{red}}^2} = \frac{4}{2.92} = 1.37$$
First, calculate a few new terms $w_k$ and $\bar{X}'$, where $w_k$ refers to \textit{weight for each group} and $\bar{X}'$ refers to \textit{weighted grand mean}.

\[
w_k = \frac{n_k}{S_k^2}
\]

\[
\bar{X}' = \frac{\sum (w_k \bar{X}_k)}{\sum w_k}
\]

For example, the Red group’s weight would be the number of individuals in the Red group divided by the variance of the Red group:

\[
w_{red} = \frac{n_{red}}{S_{red}^2} = \frac{4}{2.92} = 1.37
\]
Next, calculate the $F$ or rather $F''$ as it is the correct symbol under Welch’s procedure.
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\[ F'' = \frac{\sum w_k (\bar{x}_k - \bar{x}')^2}{1 + \left[ \frac{2(k-2)}{k^2 - 1} \right] \times \left[ \sum \left( \frac{1}{n_k - 1} \right) \left( 1 - \frac{w_k}{\sum w_k} \right)^2 \right]} \]
Next, calculate the $df'$
Next, calculate the $df'$

$$df' = \frac{k^2 - 1}{3 \sum \left( \frac{1}{n_k - 1} \right) \left( 1 - \frac{w_k}{\sum w_k} \right)^2}$$
Next, calculate the $df'$

$$
df' = \frac{k^2 - 1}{3 \sum \left(\frac{1}{n_k - 1}\right) \left(1 - \frac{w_k}{\sum w_k}\right)^2}
$$

Now use $df'$ and $k - 1 = df_b$ to find the critical value in the standard $F$ table using $df_b$ as the numerator and $df'$ as the denominator.
Next, calculate the $df'$

\[
df' = \frac{k^2 - 1}{3 \sum \left( \frac{1}{n_k - 1} \right) \left( 1 - \frac{w_k}{\sum w_k} \right)^2}
\]

Now use $df'$ and $k - 1 = df_b$ to find the critical value in the standard $F$ table using $df_b$ as the numerator and $df'$ as the denominator.

See the Supplemental Handout for a complete example of the Welch Robust Procedure.
Module 9 covered the following topics:

- Introduction to One-way ANOVA
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- Introduction to One-way ANOVA
- One-Way ANOVA Hypothesis Testing
Module 9 covered the following topics:

- Introduction to One-way ANOVA
- One-Way ANOVA Hypothesis Testing
  - Planned Comparisons
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- Introduction to One-way ANOVA
- One-Way ANOVA Hypothesis Testing
  - Planned Comparisons
  - Post-hoc Testing

A firm understanding of the topics covered here and previously will be necessary for understanding future topics.
Module 9 covered the following topics:

- Introduction to One-way ANOVA
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Module 9 covered the following topics:

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  - Confidence Intervals
Module 9 covered the following topics:

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  - Planned Comparisons
  - Post-hoc Testing
  - Effect Size
  - Confidence Intervals
- Graphing

Additional Considerations:
- Power
- Welch Robust Procedure

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A firm understanding of the topics covered here and previously will be necessary for understanding future topics.
This concludes Module 9

Next time Module 10.

- Next time we’ll begin covering Regression.
- Until next time; have a nice day.

These slides initially created on: October 15, 2010
These slides last updated on: October 21, 2010

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