MULTIPLE REGRESSION IN BEHAVIORAL RESEARCH

EXPLANATION AND PREDICTION

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CHAPTER

3

Regression Diagnostics

Merits of most regression diagnostics can be especially appreciated in multiple regression analysis (i.e., analysis with more than one independent variable). Some diagnostics are applicable only in this case. Further, familiarity with matrix algebra and analysis by computer are essential for the application and understanding of most diagnostics. Nevertheless, a rudimentary introduction in the context of simple regression analysis should prove helpful because the calculations involved are relatively simple, requiring neither matrix operations nor computer analysis. After introducing computer programs and basic notions of matrix algebra (Chapters 4 and 5), I elaborate and expand on some topics I introduce here.[1] The present introduction is organized under two main headings: "Outliers" and "Influence Analysis."[2]

OUTLIERS

As the name implies, an outlier is a data point distinct or deviant from the rest of the data. Of factors that may give rise to outliers, diverse errors come readily to mind. Thus, an outlier may be a result of a recording or an input error, measurement errors, the malfunctioning of an instrument, or inappropriate instructions in the administration of a treatment, to name but some. Detecting errors and correcting them, or discarding subjects when errors in their scores are not correctable, are the recommended strategies in such instances.

Outliers may occur in the absence of errors. In essence, these are "true" outliers, as contrasted with "false" ones arising from errors of the kind I discussed in the preceding paragraph. It is outliers not due to discernable errors that are of interest for what they may reveal, among other things, about (1) the model being tested, (2) the possible violation of assumptions, and (3) observations that have undue influence on the results.[3] This is probably what Kruskal (1988) had in mind when he asserted that "investigation of the mechanism for outlying may be far more important than the original study that led to the outlier" (p. 929).

[1] An advanced review of topics presented in this chapter is given by Chatterjee and Hadi (1986a) and is followed by comments by some leading authorities. See also Hoaglin (1992) for a very good explication of diagnostics.

[2] I do not present here diagnostic approaches addressed to issues of collinearity (see Chapter 10), as they are only relevant for the case of multiple regression analysis.

[3] As I explain in the next section, an influential observation is a special case of an outlier.

Individuals with a unique attribute, or a unique combination of attributes, may react uniquely to a treatment making them stand out from the rest of the group. Discovery of such occurrences may lead to new insights into the phenomenon under study and to the designing of research to explore and extend such insights.

DETECTION OF OUTLIERS

Procedures for the detection of outliers rely almost exclusively on the detection of extreme residuals, so much so that the two are used interchangeably by some authors and researchers. Using the outlier concept in a broader sense of a deviant case, it is possible for it to be associated with a small residual, even one equal to zero. Such outliers may become evident when studying influence analysis—a topic I present in the next section. In what follows, I present three approaches to the detection of outliers based on residual analysis: (1) standardized residuals, (2) studentized residuals, and (3) studentized deleted residuals.

Standardized Residuals (ZRESID)

I introduced standardized residuals in Chapter 2—see (2.34) and the discussion related to it. Various authors have suggested that standardized residuals greater than 2 in absolute value (i.e., $z > |2.0|$) be scrutinized. Notice that large standardized residuals serve to alert the researcher to study them; *not* to automatically designate the points in question as outliers. As in most other matters, what counts is informed judgment. The same is true of studentized and studentized deleted residuals, which I discuss later in the chapter.

To illustrate the calculation of the various indices presented here, I will use data from the numerical example I introduced in Chapter 2 (Table 2.1). For convenience, I repeat the data from Table 2.1 in the first two columns of Table 3.1. Also repeated in the table, in the column labeled RESID, are residuals I took from Table 2.2.

As an example, I will calculate the standardized residual for the last subject in Table 3.1. This subject's residual is –2.80. For the data under consideration, $s_{y.x} = 2.446$ (see Chapter 2, for calculations). Dividing the residual by 2.446 yields a standardized residual of –1.1447.

Standardized residuals for the rest of the subjects, reported in Table 3.1 in the column labeled ZRESID, were similarly calculated. As you can see, none of the standardized residuals is greater than $|2.0|$. Had standardized residuals been used for detection of outliers, it would have been plausible to conclude that there are no outliers in the data under consideration.

Studentized Residuals (SRESID)

Calculation of standardized residuals is based on the generally untenable assumption that all residuals have the same variance. To avoid making this assumption, it is suggested that SRESIDs be used instead. This is accomplished by dividing each residual by its estimated standard deviation, which for simple regression analysis is

$$s_{e_i} = s_{y.x}\sqrt{1-\left[\frac{1}{N}+\frac{(X_i-\bar{X})^2}{\Sigma x^2}\right]} \tag{3.1}$$

Note that the standard deviation of a residual is obtained by multiplying the standard error of estimate ($s_{y.x}$)—used above as the denominator for standardized residuals—by the term under

Table 3.1 Residual Analysis for Data of Table 2.1

X	*Y*	*RESID*	*ZRESID*	*SRESID*	*SDRESID*
1	3	−2.80	−1.1447	−1.2416	−1.2618
1	5	−.80	−.3271	−.3547	−.3460
1	6	.20	.0818	.0887	.0862
1	9	3.20	1.3082	1.4190	1.4632
2	4	−2.55	−1.0425	−1.0839	−1.0895
2	6	−.55	−.2248	−.2338	−.2275
2	7	.45	.1840	.1913	.1861
2	10	3.45	1.4104	1.4665	1.5188
3	4	−3.30	−1.3491	−1.3841	−1.4230
3	6	−1.30	−.5315	−.5453	−.5343
3	8	.70	.2862	.2936	.2860
3	10	2.70	1.1038	1.1325	1.1420
4	5	−3.05	−1.2469	−1.2965	−1.3232
4	7	−1.05	−.4293	−.4463	−.4362
4	9	.95	.3884	.4038	.3942
4	12	3.95	1.6148	1.6790	1.7768
5	7	−1.80	−.7359	−.7982	−.7898
5	10	1.20	.4906	.5321	.5212
5	12	3.20	1.3082	1.4190	1.4633
5	6	−2.80	−1.1447	−1.2416	−1.2618

NOTE: *X* and *Y* were taken from Table 2.1.
RESID = residual (taken from Table 2.2)
ZRESID = standardized residual
SRESID = studentized residual
SDRESID = studentized deleted residual
See text for explanations.

the radical. Examine the latter and notice that the more X_i deviates from the mean of *X*, the smaller the standard error of the residual; hence the larger the studentized residual. As I show in the "Influence Analysis" section of this chapter, the term in the brackets (i.e., that subtracted from 1) is referred to as *leverage* and is symbolized as h_i.[4]

For illustrative purposes, I will apply (3.1) to the last subject of Table 3.1. For the data of Table 3.1, $\bar{X} = 3.00$ and $\Sigma x^2 = 40$ (see Chapter 2 for calculations). Hence,

$$s_{e_i} = 2.446\sqrt{1 - \left[\frac{1}{20} + \frac{(5 - 3.0)^2}{40}\right]} = 2.2551$$

Dividing the residual (−2.80) by its standard deviation (2.2551), SRESID for the last subject is −1.2416. Note that subjects having the same *X* have an identical standard error of residual. For example, the standard error of the residual for the last four subjects is 2.2551. Dividing these subjects' residuals by 2.2551 yields their SRESIDs. Studentized residuals for all the subjects in the example under consideration are reported in Table 3.1 under SRESID.

[4]The *h* stands for the so-called hat matrix, and *i* refers to the *i*th diagonal element of this matrix. If you are unfamiliar with matrix terminology, don't worry about it. I explain it in subsequent chapters (especially Chapter 6). I introduced the term here because it affords a simpler presentation of some subsequent formulas in this section and in the "Influence Analysis" section presented in this chapter.

When the assumptions of the model are reasonably met, SRESIDs follow a t distribution with $N - k - 1$ *df*, where N = sample size, k = number of independent variables. For the present example, $df = 18$ (20 − 1 − 1). It should be noted that the t's are not independent. This, however, is not a serious drawback, as the usefulness of the t's lies not so much in their use for tests of significance of residuals but as indicators of relatively large residuals whose associated observations deserve scrutiny.

The SRESIDs I discussed thus far are referred to by some authors (e.g., Cook & Weisberg, 1982, pp. 18–20) as "internally studentized residuals," to distinguish them from "externally studentized residuals." The distinction stems from the fact that $s_{y.x}$ used in the calculation of internally studentized residuals is based on the data for *all* the subjects, whereas in the case of externally studentized residuals $s_{y.x}$ is calculated after excluding the individual whose studentized residual is being sought (see the next section).

Studentized Deleted Residuals (SDRESID)

The standard error of SDRESID is calculated in a manner similar to (3.1), except that the standard error of estimate is based on data from which the subject whose studentized deleted residual is being sought was excluded. The reasoning behind this approach is that to the extent that a given point constitutes an outlier, its retention in the analysis would lead to upward bias in the standard error of estimate ($s_{y.x}$), thereby running the risk of failing to identify it as an outlier. Accordingly, the standard error of a deleted residual is defined as

$$s_{e(i)} = s_{y.x(i)} \sqrt{1 - \left[\frac{1}{N} + \frac{(X_i - \bar{X})^2}{\Sigma x^2}\right]} \tag{3.2}$$

where $s_{e(i)}$ = standard error of residual for individual i, who has been excluded from the analysis; and $s_{y.x(i)}$ = standard error of estimate based on data from which i was excluded. Dividing i's residual by this standard error yields a SDRESID, which, as I stated previously, is also called an externally studentized residual.

For illustrative purposes, I will calculate SDRESID for the last subject of Table 3.1. This requires that the subject in question be deleted and a regression analysis be done to obtain the standard error of estimate.[5] Without showing the calculations, the standard error of estimate based on the data from which the last subject was deleted (i.e., an analysis based on the first 19 subjects) is 2.407. Applying (3.2),

$$s_{e_{(1)}} = 2.407 \sqrt{1 - \left[\frac{1}{20} + \frac{(5 - 3.0)^2}{40}\right]} = 2.2190$$

Dividing the last subject's residual (−2.80) by this standard error yields a SDRESID of −1.2618.

As you can see, application of (3.2) for all the subjects would entail 20 regression analyses, in each of which one subject is deleted. Fortunately, formulas obviating the need for such laboriously repetitious calculations are available.[6] Following are two alternative approaches to the calculation of SDRESID based on results of an analysis in which all the subjects were included.

$$SDRESID_{(i)} = e_i \sqrt{\frac{N - k - 2}{ss_{\text{res}}(1 - h_i) - e_i^2}} \tag{3.3}$$

[5]Later, I give formulas that obviate the need to do a regression analysis from which the subject in question was excluded.

[6]As I show in Chapter 4, current computer programs for regression analysis include extensive diagnostic procedures.

where $SDRESID_{(i)}$ = studentized deleted residual for subject i; e_i = residual for subject i; N = sample size; k = number of independent variables; ss_{res} = residual sum of squares from the analysis in which *all* the subjects were included; and $h_i = 1/N + (X_i - \bar{X})^2/\Sigma x^2$—see (3.1) and Footnote 4.

Using (3.3), I will calculate SDRESID for the last subject in the present example (using the data from Table 3.1). Recall that $N = 20$, and $k = 1$. From earlier calculations, $e_{20} = -2.80$; the mean of $X = 3.0$; $ss_{res} = 107.70$. Hence,

$$SDRESID_{(20)} = -2.80\sqrt{\frac{20 - 1 - 2}{107.70\,(1 - .15) - (-2.80)^2}} = -1.2618$$

which agrees with the value I obtained previously. Similarly, I calculated SDRESIDs for the rest of the subjects. I reported them in Table 3.1 under SDRESID.[7]

Having calculated studentized residuals—as I did earlier and reported under SRESID in Table 3.1—SDRESIDs can also be calculated as follows:

$$SDRESID_{(i)} = SRESID_i\sqrt{\frac{N - k - 2}{N - k - 1 - SRESID_i^2}} \tag{3.4}$$

where all the terms were defined earlier.

Using (3.4), I will calculate SDRESID for the last subject of Table 3.1. From earlier calculations (see also Table 3.1), $SRESID_{20} = -1.2416$. Hence,

$$SDRESID_{(20)} = -1.2416\sqrt{\frac{20 - 1 - 2}{20 - 1 - 1 - (-1.2416)^2}} = -1.2619$$

which is, within rounding, the same value I obtained earlier.

The SDRESID is distributed as a t distribution with $N - k - 2$ *df.* As with ZRESID and SRESID, it is generally used not for tests of significance but for identifying large residuals, alerting the user to examine the observations associated with them.

INFLUENCE ANALYSIS

Although it has been recognized for some time that certain observations have greater influence on regression estimates than others, it is only in recent years that various procedures were developed for identifying influential observations. In their seminal work on influence analysis, Belsley, Kuh, and Welsch (1980) defined an influential observation as

> one which, either individually or together with several other observations, has a demonstrably larger impact on the calculated values of various estimates (coefficients, standard errors, *t*-values, etc.) than is the case for most of the other observations. (p. 11)

As I illustrate later in this chapter, an outlier (see the preceding section) is not necessarily an influential observation. Rather, "an influential case is a special kind of outlier" (Bollen & Jackman, 1985, p. 512). As with outliers, greater appreciation of the role played by influential observations can be gained in the context of multiple regression analysis. Nevertheless, I introduce this topic here for the same reasons I introduced outliers earlier, namely, in simple regression

[7]For the present data, SDRESIDs differ little from SRESIDs. In the next section, I give an example where the two differ considerably.

analysis the calculations are simple, requiring neither matrix operations nor computer analysis. Later in the text (especially Chapter 6), I show generalizations to multiple regression analysis of indices presented here.

LEVERAGE

As the name implies, an observation's undue influence may be likened to the action of a lever providing increased power to pull the regression line, say, in a certain direction. In simple regression analysis, leverage can be calculated as follows:

$$h_i = \frac{1}{N} + \frac{(X - \bar{X})^2}{\Sigma x^2} \tag{3.5}$$

As I pointed out earlier—see (3.1) and the discussion related to it—h_i refers to the ith diagonal element of the so-called hat matrix (see Chapter 6). Before applying (3.5) to the numerical example under consideration, I will list several of its properties.

1. Leverage is a function solely of scores on the independent variable(s). Thus, as I show in the next section, a case that may be influential by virtue of its status on the dependent variable will not be detected as such on the basis of its leverage.
2. Other things equal, the larger the deviation of X_i from the mean of X, the larger the leverage. Notice that leverage is at a minimum ($1/N$) when X_i is equal to the mean of X.
3. The maximum value of leverage is 1.
4. The average leverage for a set of scores is equal to $(k + 1)/N$, where k is the number of independent variables.

In light of these properties of leverage, Hoaglin and Welsch (1978, p. 18) suggested that, as a rule of thumb, $h_i > 2(k + 1)/N$ be considered high (but see Velleman & Welsch, 1981, pp. 234–235, for a revision of this rule of thumb in light of N and the number of independent variables). Later in this chapter, I comment on rules of thumb in general and specifically for the detection of outliers and influential observations and will therefore say no more about this topic here.

For illustrative purposes, I will calculate h_{20} (leverage for the last subject of the data in Table 3.1). Recalling that $N = 20$, $X_{20} = 5$, $\bar{X} = 3$, $\Sigma x^2 = 40$,

$$h_{20} = \frac{1}{20} + \frac{(5-3)^2}{40} = .15$$

Leverage for subjects having the same X is, of course, identical. Leverages for the data of Table 3.1 are given in column (1) of Table 3.2, from which you will note that all are relatively small, none exceeding the criterion suggested earlier.

To give you a feel for an observation with high leverage, and how such an observation might affect regression estimates, assume for the last case of the data in Table 3.1 that $X = 15$ instead of 5. This may be a consequence of a recording error or it may truly be this person's score on the independent variable. Be that as it may, after the change, the mean of X is 3.5, and $\Sigma x^2 = 175.00$ (you may wish to do these calculations as an exercise). Applying now (3.5), leverage for the changed case is .81 (recall that maximum leverage is 1.0).

Table 3.2 Influence Analysis for Data of Table 3.1

(1) *h* *Leverage*	*(2)* *Cook's* *D*	*(3)* *a* *DFBETA*	*(4)* *b* *DFBETA*	*(5)* *a* *DFBETAS*	*(6)* *b* *DFBETAS*
.15	.13602	−.65882	.16471	−.52199	.43281
.15	.01110	−.18824	.04706	−.14311	.11866
.15	.00069	.04706	−.01176	.03566	−.02957
.15	.17766	.75294	−.18824	.60530	−.50189
.07	.04763	−.34459	.06892	−.27003	.17912
.07	.00222	−.07432	.01486	−.05640	.03741
.07	.00148	.06081	−.01216	.04612	−.03059
.07	.08719	.46622	−.09324	.37642	−.24969
.05	.05042	−.17368	.00000	−.13920	.00000
.05	.00782	−.06842	.00000	−.05227	.00000
.05	.00227	.03684	.00000	.02798	.00000
.05	.03375	.14211	.00000	.11171	.00000
.07	.06814	.08243	−.08243	.06559	−.21754
.07	.00808	.02838	−.02838	.02162	−.07171
.07	.00661	−.02568	.02568	−.01954	.06481
.07	.11429	−.10676	.10676	−.08807	.29210
.15	.05621	.21176	−.10588	.16335	−.27089
.15	.02498	−.14118	.07059	−.10781	.17878
.15	.17766	−.37647	.18824	−.30265	.50189
.15	.13602	.32941	−.16471	.26099	−.43281

NOTE: The data, originally presented in Table 2.1, were repeated in Table 3.1. I discuss Column (2) under Cook's *D* and Columns (3) through (6) under DFBETA. a = intercept.

Using the data in Table 3.1, change X for the last case to 15, and do a regression analysis. You will find that

$$Y' = 6.96 + .10X$$

In Chapter 2—see the calculations following (2.9)—the regression equation for the original data was shown to be

$$Y' = 5.05 + .75X$$

Notice the considerable influence the change in one of the X's has on both the intercept and the regression coefficient (incidentally, r^2 for these data is .013, as compared with .173 for the original data). Assuming one could rule out errors (e.g., of recording, measurement, see the earlier discussion of this point), one would have to come to grips with this finding. Issues concerning conclusions that might be reached, and actions that might be taken, are complex. At this stage, I will give only a couple of examples.

Recall that I introduced the numerical example under consideration in Chapter 2 in the context of an experiment. Assume that the researcher had intentionally exposed the last subject to $X = 15$ (though it is unlikely that only one subject would be used). A possible explanation for the undue influence of this case might be that the regression of Y on X is curvilinear rather than linear. That is, the last case seems to change a linear trend to a curvilinear one (*but see the caveats that follow;* note also that I present curvilinear regression analysis in Chapter 13).

Assume now that the data of Table 3.1 were collected in a nonexperimental study and that errors of recording, measurement, and the like were ruled out as an explanation for the last person's X score being so deviant (i.e., 15). One would scrutinize attributes of this person in an attempt to discern what it is that makes him or her different from the rest of the subjects. As an admittedly unrealistic example, suppose that it turns out that the last subject is male, whereas the rest are females. This would raise the possibility that the status of males on X is considerably higher than that of females. Further, that the regression of Y on X among females differs from that among males (I present comparison of regression equations for different groups in Chapter 14).

Caveats. *Do not place too much faith in speculations such as the preceding.* Needless to say, one case does not a trend make. At best, influential observations should serve as clues. Whatever the circumstances of the study, and whatever the researcher's speculations about the findings, two things should be borne in mind.

1. Before accepting the findings, it is necessary to ascertain that they are replicable in newly designed studies. Referring to the first illustration given above, this would entail, among other things, exposure of more than one person to the condition of $X = 15$. Moreover, it would be worthwhile to also use intermediate values of X (i.e., between 5 and 15) so as to be in a position to ascertain not only whether the regression is curvilinear, but also the nature of the trend (e.g., quadratic or cubic; see Chapter 13). Similarly, the second illustration would entail, among other things, the use of more than one male.
2. Theoretical considerations should play the paramount role in attempts to explain the findings.

Although, as I stated previously, leverage is a property of the scores on the independent variable, the extent and nature of the influence a score with high leverage has on regression estimates depend also on the Y score with which it is linked. To illustrate this point, I will introduce a different change in the data under consideration. Instead of changing the last X to 15 (as I did previously), I will change the one before the last (i.e., the 19th subject) to 15.

Leverage for this score is, of course, the same as that I obtained above when I changed the last X to 15 (i.e., .81). However, the regression equation for these data differs from that I obtained when I changed the last X to 15. When I changed the last X to 15, the regression equation was

$$Y' = 6.96 + .10X$$

Changing the X for the 19th subject to 15 results in the following regression equation:

$$Y' = 5.76 + .44X$$

Thus, the impact of scores with the same leverage may differ, depending on the dependent-variable score with which they are paired. You may find it helpful to see why this is so by plotting the two data sets and drawing the regression line for each. Also, if you did the regression calculations, you would find that $r^2 = .260$ when the score for the 19th subject is changed to 15, as contrasted with $r^2 = .013$ when the score for the 20th subject is changed to 15. Finally, the residual and its associated transformations (e.g., standardized) are smaller for the second than for the first change:

	X	Y	Y'	$Y - Y'$	*ZRESID*	*SRESID*	*SDRESID*
20th subject	15	6	8.4171	−2.4171	−.9045	−2.0520	−2.2785
19th subject	15	12	12.3600	−.3600	−.1556	−.3531	−.3443

Based on residual analysis, the 20th case might be deemed an outlier, whereas the 19th would not be deemed thus.

COOK'S D

Earlier, I pointed out that leverage cannot detect an influential observation whose influence is due to its status on the dependent variable. By contrast, Cook's (1977, 1979) D (distance) measure is designed to identify an influential observation whose influence is due to its status on the independent variable(s), the dependent variable, or both.

$$D_i = \left[\frac{SRESID_i^2}{k+1}\right]\left[\frac{h_i}{1-h_i}\right] \tag{3.6}$$

where SRESID = studentized residual (see the "Outliers" section presented earlier in this chapter); h_i = leverage (see the preceding); and k = number of independent variables. Examine (3.6) and notice that D will be large when SRESID is large, leverage is large, or both.

For illustrative purposes, I will calculate D for the last case of Table 3.1. $\text{SRESID}_{20} = -1.2416$ (see Table 3.1); $h_{20} = .15$ (see Table 3.2); and $k = 1$. Hence,

$$D_{20} = \left[\frac{-1.2416^2}{1+1}\right]\left[\frac{.15}{1-.15}\right] = .1360$$

D's for the rest of the data of Table 3.1 are given in column (2) of Table 3.2.

Approximate tests of significance for Cook's D are given in Cook (1977, 1979) and Weisberg (1980, pp. 108–109). For diagnostic purposes, however, it would suffice to look for relatively large D values, that is, one would look for relatively large gaps between D for a given observation and D's for the rest of the data. Based on our knowledge about the residuals and leverage for the data of Table 3.1, it is not surprising that all the D's are relatively small, indicating the absence of influential observations.

It will be instructive to illustrate a situation in which leverage is relatively small, implying that the observation is not influential, whereas Cook's D is relatively large, implying that the converse is true. To this end, change the last observation so that $Y = 26$. As X is *unchanged* (i.e., 5), the leverage for the last case is .15, as I obtained earlier. Calculate the regression equation, SRESID, and Cook's D for the last case. Following are some of the results you will obtain:

$$Y' = 3.05 + 1.75X$$

$$SRESID_{20} = 3.5665;\ h_{20} = .15;\ k = 1$$

Notice the changes in the parameter estimates resulting from the change in the Y score for the 20th subject.[8] Applying (3.6),

$$D_{20} = \left[\frac{3.5665^2}{1+1}\right]\left[\frac{.15}{1-.15}\right] = 1.122$$

If you were to calculate D's for the rest of the data, you would find that they range from .000 to .128. Clearly, there is a considerable gap between D_{20} and the rest of the D's. To reiterate, sole reliance on leverage would lead to the conclusion that the 20th observation is not influential, whereas the converse conclusion would be reached based on the D.

[8]Earlier, I pointed out that SRESID (studentized residual) and SDRESID (studentized deleted residual) may differ considerably. The present example is a case in point, in that $\text{SDRESID}_{20} = 6.3994$.

I would like to make two points about my presentation of influence analysis thus far.

1. My presentation proceeded backward, so to speak. That is, I examined consequences of a change in an X or Y score on regression estimates. Consistent with the definition of an influential observation (see the preceding), a more meaningful approach would be to study changes in parameter estimates that would occur because of deleting a given observation.
2. Leverage and Cook's D are global indices, signifying that an observation may be influential, but not revealing the effects it may have on specific parameter estimates.

I now turn to an approach aimed at identifying effects on specific parameter estimates that would result from the deletion of a given observation.

DFBETA

DFBETA$_{j(i)}$ indicates the change in j (intercept or regression coefficient) as a consequence of deleting subject i.[9] As my concern here is with simple regression analysis—consisting of two parameter estimates—it will be convenient to use the following notation: DFBETA$_{a(i)}$ will refer to the change in the intercept (a) when subject i is deleted, whereas DFBETA$_{b(i)}$ will refer to the change in the regression coefficient (b) when subject i is deleted.

To calculate DFBETA for a given observation, then, delete it, recalculate the regression equation, and note changes in parameter estimates that have occurred. For illustrative purposes, delete the last observation in the data of Table 3.1 and calculate the regression equation. You will find it to be

$$Y' = 4.72 + .91X$$

Recall that the regression equation based on all the data is

$$Y' = 5.05 + .75X$$

Hence, DFBETA$_{a(20)}$ = .33 (5.05 − 4.72), and DFBETA$_{b(20)}$ = −.16 (.75 − .91). Later, I address the issue of what is to be considered a large DFBETA, hence identifying an influential observation.

The preceding approach to the calculation of DFBETAs is extremely laborious, requiring the calculation of as many regression analyses as there are subjects (20 for the example under consideration). Fortunately, an alternative approach based on results obtained from a single regression analysis in which all the data are used is available. The formula for DFBETA for a is

$$DFBETA_{a(i)} = a - a(i) = \left[\left(\frac{\Sigma X^2}{N\Sigma X^2 - (\Sigma X)^2}\right) + \left(\frac{-\Sigma X}{N\Sigma X^2 - (\Sigma X)^2}\right)X_i\right]\frac{e_i}{1 - h_i} \quad (3.7)$$

where N = number of cases; ΣX^2 = sum of squared raw scores; ΣX = sum of raw scores; $(\Sigma X)^2$ = square of the sum of raw scores; e_i = residual for subject i; and h_i = leverage for subject i. Earlier,

[9]DF is supposed to stand for the difference between the estimated statistic with and without a given case. I said "supposed," as initially the prefix for another statistic suggested by the originators of this approach (Belsley et al., 1980) was DI, as in DIFFITS, which was then changed to DFFITS and later to DFITS (see Welsch, 1986, p. 403). Chatterjee and Hadi (1986b) complained about the "computer-speak (à la Orwell)," saying, "We aesthetically rebel against DFFIT, DFBETA, etc., and have attempted to replace them by the last name of the authors according to a venerable statistical tradition" (p. 416). Their hope that "this approach proves attractive to the statistical community" (p. 416) has not materialized thus far.

I calculated all the preceding terms. The relevant sum and sum of squares (see Table 2.1 and the presentation related to it) are

$$\Sigma X = 60 \qquad \Sigma X^2 = 220$$

$N = 20$. Residuals are given in Table 3.1, and leverages in Table 3.2.

For illustrative purposes, I will apply (3.7) to the last (20th) case, to determine the change in a that would result from its deletion.

$$DFBETA_{a(20)} = a - a(20) = \left[\left(\frac{220}{(20)(220)-(60)^2}\right) + \left(\frac{-60}{(20)(220)-(60)^2}\right)5\right]\frac{-2.8}{1-.15} = .32941$$

which agrees with the result I obtained earlier.

The formula for DFBETA for b is

$$DFBETA_{b(i)} = b - b(i) = \left[\left(\frac{-\Sigma X}{N\Sigma X^2-(\Sigma X)^2}\right) + \left(\frac{N}{N\Sigma X^2-(\Sigma X)^2}\right)X_i\right]\frac{e_i}{1-h_i} \tag{3.8}$$

where the terms are as defined under (3.7). Using the results given in connection with the application of (3.7),

$$DFBETA_{b(20)} = b - b(20) = \left[\left(\frac{-60}{(20)(220)-(60)^2}\right) + \left(\frac{20}{(20)(220)-(60)^2}\right)5\right]\frac{-2.8}{1-.15} = -.16471$$

which agrees with the value I obtained earlier.

To repeat, DFBETAs indicate the change in the intercept and the regression coefficient(s) resulting from the deletion of a given subject. Clearly, having calculated DFBETAs, calculation of the regression equation that would be obtained as a result of the deletion of a given subject is straightforward. Using, as an example, the DFBETAs I calculated for the last subject (.33 and −.16 for a and b, respectively), and recalling that the regression equation based on all the data is $Y' = 5.05 + .75X$,

$$a = 5.05 - .33 = 4.72$$

$$b = .75 - (-.16) = .91$$

Above, I obtained the same values when I did a regression analysis based on all subjects but the last one.

Using (3.7) and (3.8), I calculated DFBETAs for all the subjects. They are given in columns (3) and (4) of Table 3.2.

Standardized DFBETA

What constitutes a large DFBETA? There is no easy answer to this question, as it hinges on the interpretation of regression coefficients—a topic that will occupy us in several subsequent chapters. For now, I will only point out that the size of the regression coefficient (hence a change in it) is affected by the scale of measurement used. For example, using feet instead of inches to measure X will yield a regression coefficient 12 times larger than one obtained for inches, though the nature of the regression of Y on X will, of course, not change.[10]

In light of the preceding, it was suggested that DFBETA be standardized, which for a is accomplished as follows:

[10]It is for this reason that some researchers prefer to interpret standardized regression coefficients or beta weights—a topic I discuss in detail in Chapters 4 and 10.

$$DFBETAS_{a(i)} = \frac{DFBETA_{a(i)}}{\sqrt{MSR_{(i)}\left[\frac{\Sigma X^2}{N\Sigma X^2 - (\Sigma X)^2}\right]}} \quad (3.9)$$

where DFBETAS = standardized DFBETA;[11] and $MSR_{(i)}$ = mean square residual when subject i is deleted. The rest of the terms were defined earlier.

If, as I suggested earlier, you did a regression analysis in which the last subject was deleted, you would find that $MSR_{(20)} = 5.79273$. Hence,

$$DFBETAS_{a(20)} = \frac{.32941}{\sqrt{5.79273\left[\frac{220}{(20)(220) - (60)^2}\right]}} = .26099$$

The formula for standardizing DFBETA for b is

$$DFBETAS_{b(i)} = \frac{DFBETA_{b(i)}}{\sqrt{MSR_{(i)}\left[\frac{N}{N\Sigma X^2 - (\Sigma X)^2}\right]}} \quad (3.10)$$

Applying (3.10) to the 20th case,

$$DFBETAS_{b(20)} = \frac{-.16471}{\sqrt{5.79273\left[\frac{20}{(20)(220) - (60)^2}\right]}} = -.43282$$

Notice that $MSR_{(i)}$ in the denominator of (3.9) and (3.10) is based on an analysis in which a given subject is deleted. Hence, as many regression analyses as there are subjects would be required to calculate DFBETAS for all of them. To avoid this, $MSR_{(i)}$ can be calculated as follows:

$$MSR_{(i)} = \frac{ss_{res} - \frac{(e_i)^2}{1 - h_i}}{N - k - 1 - 1} \quad (3.11)$$

For comparative purposes, I will apply (3.11) to the 20th subject. $e_{(20)} = -2.8$ (see Table 3.1); $h_{(20)} = .15$ (see Table 3.2); $ss_{res} = 107.70$ (see earlier calculations). $N = 20$ and $k = 1$. Therefore,

$$MSR_{(20)} = \frac{107.70 - \frac{(-2.8)^2}{1 - .15}}{20 - 1 - 1 - 1} = 5.79273$$

which agrees with the value I obtained earlier. Using (3.7) through (3.11), I calculated DFBETAS for all the subjects in the example under consideration (i.e., Table 3.1) and reported the results in columns (5) and (6) of Table 3.2.

In line with the recommendation that DFBETAS (standardized) be used instead of DFBETA (nonstandardized) for interpretive purposes (see preceding), criteria for what is to be considered a "large" DFBETAS have been proposed. Not surprisingly, there is no consensus on this point. Following are some examples of cutoffs that have been proposed.

Belsley et al. (1980) suggested, "as a first approximation," an "*absolute cutoff*" of 2 (p. 28). They went on to suggest that, because DFBETAS is affected by sample size, $2/\sqrt{n}$ serve as a

[11]For consistency with the literature on this topic, I use DFBETAS, although something like STDFBETA would be less prone to confuse.

"*size-adjusted cutoff*" (p. 28), when small samples are used. Neter, Wasserman, and Kutner (1989, p. 403), on the other hand, recommended that $2/\sqrt{n}$ serve as a cutoff for "large data sets," whereas 1 serve as a cutoff for "small to medium-size data sets." Finally, Mason, Gunst, and Hess (1989, p. 520) proposed $3/\sqrt{n}$ as a general cutoff.

Recalling that $N = 20$ for the example under consideration, following Belsley et al., the size-adjusted cutoff is .45, whereas following Mason et al. the cutoff is .67. Examine columns (5) and (6) of Table 3.2 and notice that a few of the DFBETASs are slightly larger than the size-adjusted cutoff proposed by Belsley et al. and that none meet the criteria proposed by Neter et al. or Mason et al. In sum, it is safe to assume that most researchers would conclude that none of the DFBETASs in the numerical example under consideration are "large."

Before I comment generally on criteria and rules of thumb, I will use an additional example to illustrate: (1) the value of DFBETA in pinpointing changes occurring as a result of the deletion of a subject and (2) that an outlier does *not* necessarily signify that the observation in question is influential. To this end, let us introduce yet another change in the data of Table 3.1. This time, change the Y for the first subject in the group whose $X = 3$ (i.e., the ninth subject) to 14 (instead of 4). Calculate the regression equation. In addition, for this subject, calculate (1) ZRESID, SRESID, and SDRESID; (2) leverage and Cook's D; (3) DFBETA (nonstandardized) and DFBETAS (standardized). Following are results you will obtain:

$$Y' = 5.55 + .75X$$

For the ninth subject,

(1)	*ZRESID*	*SRESID*	*SDRESID*
	2.2498	2.3082	2.6735
(2)	*Leverage*	*Cook's D*	
	.050	.140	
(3)	*DFBETA*	*DFBETAS*	
	a: .32632	.26153	
	b: .00000	.00000	

Beginning with the residual, note that the observation under consideration would probably be identified as an outlier, especially when it is compared with those for the rest of the data. For example, the next largest SDRESID is −1.3718.

Turning to leverage, it is clear that it is small. The same is true of D. If you were to calculate the D's for the rest of the data, you would find that they range from .000 to .149. Clearly, the D for the ninth subject is not out of line from the rest of the D's, leading to the conclusion that the ninth observation is not influential. Here, then, is an example where an observation that might be identified as an outlier would not be deemed as influential.

Examine now the DFBETA and DFBETAS and note that the deletion of the ninth subject will result in an intercept change from 5.55 to 5.22 (i.e., 5.55 − .32632). The regression coefficient will, however, *not* change as a result of the deletion of the ninth subject. Thus, the regression equation based on the data from which the ninth subject was deleted would be[12]

$$Y' = 5.22 + .75X$$

[12] If necessary, delete the ninth subject and do a regression analysis to convince yourself that this is the equation you would obtain.

It will be instructive to concentrate first on the interpretation of a change in a. Recall that a indicates the point at which the regression line intercepts the Y ordinate when $X = 0$. Stated differently, it is the predicted Y when $X = 0$. In many areas of behavioral sciences $X = 0$ is of little or no substantive meaning. Suffice it to think of X as a measure of mental ability, achievement, depression, and the like, to see why this is so. Therefore, even if the change in a was much larger than the one obtained earlier, and even if it was deemed to be large based on some criterion, it is conceivable that it would be judged not meaningful. This is not to say that one would ignore the extreme residual that would be associated with the observation in question. But this matter need not concern us here, as I addressed it earlier.

What is, however, most revealing in the present example—indeed my reason for presenting it—is the absence of change in the regression coefficient (b) as a result of deleting the ninth subject.[13] Thus, even if based on other indices (e.g., D), one was inclined to consider the ninth observation as influential, it is conceivable that focusing on the change in b, one would deem it not influential.

CRITERIA AND RULES OF THUMB

Dependence on criteria and rules of thumb in the conduct of behavioral research is so prevalent that it requires no documentation. The ubiquity of such practices is exemplified by conventions followed in connection with statistical tests of significance (e.g., Type I and Type II errors, effect size).[14]

Authors who propose criteria and rules of thumb do so, in my opinion, with the best of intentions to assist their readers to develop a "feel" for the indices in question. Notably, most stress the need for caution in resorting to criteria they propose and attempt to impress upon the reader that they are not meant to serve as substitutes for informed judgment. For instance, preceding their proposed criteria for influential observations, Belsley et al. (1980) cautioned:

> As with all empirical procedures, this question is ultimately answered by judgment and intuition in choosing reasonable cutoffs most suitable for the problem at hand, guided whenever possible by statistical theory. (p. 27)

Unfortunately, many researchers not only ignore the cautions, but also misinterpret, even misrepresent recommended guidelines.[15] Drawing attention to difficulties in interpreting outliers, Johnson (1985) bemoans the practice of treating methods for detecting them as a "technological fix," prompting "many investigators . . . to believe that statistical procedures will sort a data set into the 'good guys' and the 'bad guys'" (p. 958).

Perusal of published research reveals that many authors flaunt criteria with an air of finality and certainty. The allure of a criterion adorned by references to authorities in the field is apparently so potent as to dazzle even referees and editors of professional journals. Deleterious

[13] I suggest that you experiment by introducing other changes in Y for the same subject (e.g., make it 24, 30, or 40), and reanalyze the data. For the suggested changes, you will find the DFBETAS$_a$ becomes increasingly larger (.6623, .9027, and 1.3034, respectively), but the b is unchanged. Incidentally, the same will hold true if you changed any of the Y's whose X scores are equal to the mean of X. The main point is that when a is not substantively meaningful, neither is a change in it, whatever its size.

[14] For examples relating to measurement models, see Bollen and Lennox (1991); for examples relating to adoption of "standards" of reliability of measures, see Pedhazur and Schmelkin (1991, pp. 109–110).

[15] For some examples relating to criteria for collinearity, see Chapter 10.

consequences of this practice cannot be overestimated. The most pernicious effect of this practice is that it seems to absolve the researcher of the responsibility of making an informed interpretation and decision—actions unimaginable without thorough knowledge of the research area, an understanding of statistical and design principles, and, above all, hard thinking.

The paramount role of knowledge and judgment in deciding what is an influential observation, say, may be discerned from the last example I gave earlier. Recall that it concerned a situation in which the deletion of an observation resulted in a change in a (intercept), but not in b (regression coefficient). Clearly, a researcher whose aim is to interpret b only would not deem an observation influential, regardless of the effect its deletion would have on a.

In sum, beware of being beguiled by criteria and rules of thumb. It is only in light of various aspects of the study (e.g., cost, duration, consequences, generalizability), as well as theoretical and analytic considerations, that you can hope to arrive at meaningful statements about its findings.

A Numerical Example

Before considering remedies, I present another numerical example designed to illustrate the potential hazards of neglecting to examine one's data and of failing to apply regression diagnostics. The example is reported in Part (*a*) of Table 3.3. Included in the table are summary statistics and results of tests of statistical significance.[16] As I used a similar format in Chapter 2 (see Table 2.3), I will not explain the terms.

Table 3.3 Two Data Sets

	(*a*)		(*b*)	
	X	Y	X	Y
	2	2	2	2
	3	3	3	3
	3	1	3	1
	4	1	4	1
	4	3	4	3
	5	2	5	2
	8	8		
N:	7		6	
M:	4.14	2.86	3.50	2.00
s:	1.95	2.41	1.05	.89
r^2:	.67		.00	
a:	−1.34		2.00	
b:	1.01		.00	
ss_{reg}:	23.43		.00	
ss_{res}:	11.42		4.00	
F:	10.25 (1,5)		.00 (1,4)	
t:	3.20 (5)		.00 (4)	
p:	.02		1.00	

[16] I discuss Part (*b*) of the table later on.

Examine Part (*a*) of Table 3.3 and note that, assuming $\alpha = .05$ was selected, the regression of Y on X is statistically significant. In the absence of diagnostics, one would be inclined to conclude, among other things, that (1) about 67% of the variance in Y is accounted for by X and (2) the expected change in Y associated with a unit change in X is 1.01.

In what follows, I will scrutinize the role of the last subject in these results. The residual and some of its transformations for this subject are as follows:

RESID	*ZRESID*	*SRESID*	*SDRESID*
1.2375	.8178	1.8026	2.7249

Inspection of ZRESID and SRESID would lead to the conclusion that there is nothing distinctive about this subject, although SDRESID might raise doubt about such a conclusion.

Here now are diagnostic indices associated with the last subject:

H_7	D_7	$DFBETA_{a(7)}$	$DFBETA_{b(7)}$	$DFBETAS_{a(7)}$	$DFBETAS_{b(7)}$
.79	6.25	−3.3375	1.0125	−3.5303	4.8407

Clearly, this is an influential observation. To appreciate how influential it is, I will use DFBETAs (unstandardized) to calculate the regression equation based on the first six subjects (i.e., deleting the seventh subject).

$$a = -1.34 - (-3.34) = 2.00$$

$$b = 1.01 - 1.01 = 0$$

These statistics are reported also in Part (*b*) of Table 3.3, which consists of results of a regression analysis based on the first six subjects of Part (*a*).

The most important thing to note is that in the absence of the seventh subject, the regression of Y on X is zero ($b = 0$). At the risk of being redundant, it is noteworthy that the statistically significant and, what appeared to be, the strong regression of Y on X was due to the inclusion of a single subject.

Note that, consistent with (2.10) and the discussion related to it, when $b = 0$, the intercept (a) is equal to the mean of the dependent variable.

REMEDIES

Awareness of the existence of a problem is, needless to say, a prerequisite for attempts to do something about it. More than a decade ago, Belsley et al. (1980) observed that "[i]t is increasingly the case that the data employed in regression analysis, and on which the results are conditioned, are given only the most cursory examination for their suitability" (p. 2). Remarkable increases in availability of computers and reliance on technicians (euphemistically referred to as "consultants") to analyze one's data have greatly exacerbated this predicament.

The larger the project, the greater the likelihood for data analysis "chores" to be relegated to assistants, and the lesser the likelihood for principal investigators to examine their data. Consequently, many a researcher is unaware that "dramatic" or "puzzling" findings may be due to one or more influential observations, or that a relation they treat as linear is curvilinear, to give but two examples of illusory or delusionary findings pervading social and behavioral research literature.

Suggested Remedies

Difficulties in selecting from among indices of influential observations, and of designating observations as influential, pale in comparison to those arising concerning action to be taken when influential observations are detected. Earlier, I pointed out that when it is determined that an observation in question is due to error, the action that needs to be taken is relatively uncomplicated. It is when errors are ruled out that complications abound, as the decision regarding action to be taken is predicated on a host of theoretical and analytic considerations (e.g., model, subjects, settings). What follows is not an exhaustive presentation of remedies but a broad sketch of some, along with relevant references.

Probably the first thing that comes to mind is to delete the influential observation(s) and reanalyze the data. Nevertheless, in light of norms against "fudging" data and "dishonesty" in data analysis, the tendency to refrain from doing this is strong. I concur strongly with Judd and McClelland's (1989) cogent argument that when an influential observation(s) affects the results, it is "misleading . . . to pretend" that this is not so.

> Somehow, however, in the social sciences the reporting of results with outliers included has come to be viewed as the "honest" thing to do and the reporting of results with outliers removed is sometimes unfortunately viewed as "cheating." Although there is no doubt that techniques for outlier identification and removal can be abused, we think it far more honest to omit outliers from the analysis with the explicit admission in the report that there are some observations which we do not understand and to report a good model for those observations which we do understand. If that is not acceptable, then separate analyses, with and without the outliers included, ought to be reported so that the reader can make his or her own decision about the adequacy of the models. *To ignore outliers by failing to detect and report them is dishonest and misleading.* (pp. 231–232; see also, Fox, 1991, p. 76)

I believe that, in addition to reporting results of analyses with and without influential observations, sufficient information ought to be given (or made available on request) so that readers who so desire may reanalyze the data.

Deletion of influential observations is by no means the only suggested course of action. Among others, a transformation of one or more variables may reduce the impact of influential observations (for discussions of transformations and their role in data analysis see, among others, Atkinson, 1985, Chapters 6–9; Fox, 1984, Chapter 3; Judd & McClelland, 1989, Chapter 16; Stoto & Emerson, 1983).

Another approach is to subject the data to a robust regression method (for a review of four such methods, see Huynh, 1982; see also, Neter et al., 1989, pp. 405–407; Rousseeuw & Leroy, 1987).

CONCLUDING REMARKS

I hope that this chapter served to alert you to the importance of scrutinizing data and using regression diagnostics. In subsequent chapters, I extend and elaborate on concepts I introduced in this chapter.

In Chapter 4, which is devoted to computers and computer programs, I will use several computer programs to reanalyze some of the numerical examples I presented in Chapter 2 and in the present chapter.

STUDY SUGGESTIONS

I presented the following data in Study Suggestion 2 of Chapter 2 (recall that the second, third, and fourth pairs of columns are continuations of the first pair of columns):

X	Y	X	Y	X	Y	X	Y
2	2	4	4	4	3	9	9
2	1	5	7	3	3	10	6
1	1	5	6	6	6	9	6
1	1	7	7	6	6	4	9
3	5	6	8	8	10	4	10

For all subjects, calculate the following:

(a) ZRESID, SRESID, and SDRESID.

(b) h_i, D, DFBETA$_a$, DFBETA$_b$, DFBETAS$_a$, and DFBETAS$_b$.

NOTE: Where applicable, use intermediate results you have obtained in Study Suggestion 2 of Chapter 2.

ANSWERS

(a) ZRESID	SRESID	SDRESID
−.5822	−.6187	−.6078
−1.0291	−1.0936	−1.0999
−.6963	−.7623	−.7531
−.6963	−.7623	−.7531
.4255	.4431	.4330
−.3541	−.3646	−.3556
.6536	.6706	.6600
.2068	.2121	.2064
−.0119	−.0124	−.0121
.7677	.7910	.7825
−.8009	−.8247	−.8170
−.4682	−.4876	−.4771
−.1260	−.1298	−.1262
−.1260	−.1298	−.1262
.9958	1.0609	1.0648
.2162	.2375	.2312
−1.4570	−1.6702	−1.7657
−1.1243	−1.2352	−1.2548
1.8800	1.9357	2.1140
2.3268	2.3957	2.8211

(b) h	D	$DFBETA_a$	$DFBETA_b$	$DFBETAS_a$	$DFBETAS_b$
.1145	.025	−.23281	.03217	−.21235	.16402
.1145	.077	−.41147	.05685	−.38429	.29683
.1656	.058	−.36399	.05467	−.33389	.28033
.1656	.058	−.36399	.05467	−.33389	.28033
.0782	.008	.12553	−.01493	.11390	−.07571
.0567	.004	−.07128	.00591	−.06456	.02995
.0500	.012	.07417	.00057	.06778	.00291
.0500	.001	.02346	.00018	.02120	.00091
.0811	.000	.00073	−.00044	.00066	−.00222
.0582	.019	.02095	.01419	.01924	.07287
.0567	.020	−.16123	.01338	−.14832	.06879
.0782	.010	−.13813	.01642	−.12548	.08341
.0582	.001	−.00344	−.00233	−.00310	−.01176
.0582	.001	−.00344	−.00233	−.00310	−.01176
.1189	.076	−.15651	.05717	−.14586	.29784
.1715	.006	−.05756	.01753	−.05203	.08856
.2390	.438	.57946	−.16034	.56882	−.87989
.1715	.158	.29932	−.09115	.28233	−.48061
.0567	.113	.37844	−.03140	.38377	−.17800
.0567	.172	.46839	−.03886	.51213	−.23753